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SYNTHESIS AND ANALYSIS OF PRECISE SPACEBORNE LASER RANGING SYSTEMS

VOLUME II FINAL REPORT

MCDONNELL DOUGLAS ASTRONAUTICS COMPANY - EAST

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SYNTHESIS AND ANALYSIS OF PRECISE SPACEBORNE LASER RANGING SYSTEMS

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VOLUME II FINAL REPORT

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1. Introduction and Summary

The purpose of this study was to evaluate in detail the performance capabilities of specific SHUTTLE-based laser ranging systems, determine interface and support requirements, and to generate a preliminary design of a SHUTTLE-borne laser ranging experiment. The study was conducted in two phases, referred to as (1) experiment definition and (2) experiment design.

The goal of the experiment definition phase was to select, from the various laser ranging concepts, a viable approach to be used in the subsequent experiment design phase. The experiment objective is to make laser ranging measurements to targets on the ground, from a Shuttle based laser ranging system. These measurements are to be used to determine the relative positions of the targets with respect to each other with an overall three dimensional reconstruction accuracy better than 2 cm rms. Two basic approaches were identified. The first approach was to use relatively broad beamwidths, such that each emitted laser pulse illuminated all of the targets in a subset of the target grouping, referred to as a target grid. Thus, each pulse period would result in nearly simultaneous measurements to 2 or more targets. This approach minimizes, but does not evade, dependence on orbital mechanics to reduce the data. It was found, however, that this approach was simply not a viable concept; the energy management problem was excessively severe for target grid dimensions of the order of magnitude needed to satisfy typical measurement objectives.

The selected approach is to make the ranging measurements sequentially. However, in order to reduce the data, it is necessary to employ orbital mechanics techniques to interpolate. It was unknown, at the start of this study, just how tightly the measurements had to be interlaced, to meet the grid reconstruction accuracy goal. Consequently, a mission simulation was developed to explore the effects of various measurement strategies on the experiment design.

The simulation employed a co-variance matrix analysis technique to determine the grid reconstruction accuracy. A Kalman filter was employed to incorporate the measurements. Several measurement strategies were evaluated and a simple strategy was selected. A number of factors were found to be significant in accomplishing the desired reconstruction accuracy goals. First, we found that a Shuttle ephemeris error term coupled into the co-variance matrix of the target

grid position location unless widely spaced out-rigger targets were included in the experiment. Second, we found that at least two measurement passes were needed to reduce the reconstruction errors in all three axes. The second pass should be as nearly orthogonal to the first pass as possible. We also explored mission phasing and found that unless the pass was within a few degrees of longitude of passing directly over the target grid, the reconstruction accuracy degraded considerably. Finally, we found that the interlace requirements were not severe. A measurement strategy which resulted in a grid survey in the approach period, a grid survey near the point of closest approach, and a final grid survey in the latter portion of the viewing opportunity provided excellent results.

The modest interlace requirements do not require a high level of angular agility to satisfy the slewing and tracking requirements for the 9-target grid evaluated in the mission simulation. Thus, it was postulated that one of the stable platforms which are candidates for the role of Shuttle/Spacelab general purpose experiment pointing systems could be employed to point the laser ranging experiment transmit and receive telescopes toward the targets in the grid. The Small Instrument Pointing System (SIPS) was considered to be best suited for this purpose. Evaluation of the capabilities of the SIPS for the laser ranging experiment did not result in a clear decision between the SIPS mounted experiment and an experiment configured with a gimbaled beam steering mirror.

The study was therefore extended to include both concepts in the preliminary experiment design phase. The laser ranging systems for the two experiments were selected to be as much alike as possible. The results of the preliminary design activities are discussed in Section 3. Both designs are considered implementable with state-of-the-art components and fully capable of meeting the experiment objectives.

2. Experiment Definition

The first phase of the study was devoted to determining constraints and requirements needed to define a viable experiment. A number of concepts and alternatives were explored and are reported herein.

The experiment objective is to employ laser range measurements, made from the Shuttle borne laser ranging system to targets on the ground, to determine the relative spacings between these targets. The targets were assumed to be located roughly in a grid structure, with target arrays located at the corners of a square with one target in the middle. The grid dimension was assumed to be on the order of 10 km on a side, although the effect of varying grid size was considered. Each target in the grid was assumed to be composed of cube-corner retro-reflectors, arranged such that the effective target radar cross-section was on the order of 10^7 square meters. Grids of this sort could be used in a number of applications for various scientific or engineering purposes.

One of the most fundamental questions to be addressed was the measurement strategy to be employed. Two concepts were considered. The first concept was to employ simultaneous range measurements to two or more targets to minimize the dependency of the experiment on the Shuttle ephemeris. The second concept was to employ sequential measurements to the targets, and to reduce the data with more sophisticated signal processing. The first concept is particularly appealing since differential range measurements could be used for data processing, eliminating a number of possible bias error sources. However, as discussed in Section 2.1, simultaneous range measurements present a virtually insurmountable energy management problem for modest to large grid dimensions. The second alternative, sequential range measurements requires sophisticated signal processing to extract the desired target relative location data. The range measurements need to be interlaced, i.e., measurements to each target in the grid must be made a number of times at various look angles. One possible measurement policy is to move from target to target during each interpulse period, resulting in a maximally interlaced data set. The problem with this measurement policy is that excessive slew rates are required to accomplish this for a modest grid size. The alternative is to dwell on each target for a short period of time prior to moving to the next target. This policy maximizes the data obtained when finite slew times and settling times are considered, but

the measurements are not maximally interlaced. The effect of this measurement policy, or any other for that matter, on the achievable relative location accuracy can only be determined by simulation. This subject is addressed in Section 2.2; a simple measurement policy was shown to be quite effective, and to not require excessive angular agility to accomplish the experiment objectives.

A number of factors contribute to errors in pointing the transmitter and receiver at a specific target. These fall into two categories, location errors and attitude reference errors. Location errors include uncertainty as to the target location and uncertainties as to where the Shuttle really is.

The pointing error attributable to these sources can be reduced to negligible magnitude once several ranging measurements are used to update the experiment state vector. Pointing errors attributable to attitude reference errors tend to be nearly stationary if the Shuttle is maintained in an inertially fixed attitude during the ~5 minutes of a ranging pass. Consequently, the major pointing problem is simply to find the first target. Once the target has been detected and a few range measurements are made, the system can complete the ranging pass with open-loop pointing. Section 2.3 describes our analyses of the acquisition process devised to cope with the initial pointing uncertainties.

A number of stable platforms have been postulated to support various solar and stellar astronomy instruments for Shuttle/Spacelab experiments. The concepts differ considerably, but are intended to permit experimenters to accomplish their experiments using a general purpose, high stability, pointing system. Section 2.4 describes four of the concepts investigated, and shows that the Small Instrument Pointing System (SIPS) appears to be the most suitable general purpose pointing system for the laser ranging experiment.

2.1 Link Analysis

The performance of the laser ranging experiment is dependent on the laser energy, the pulsewidth, the target radar cross-section, the transmit beamwidth, the receiver aperture, the link geometry, the various losses encountered, the signal processing techniques employed, and the background level. In general, it is possible to select the beamwidth, the transmit energy, the receiver aperture, and the target radar cross-section to obtain sufficient

return signal energy for confident range measurements for any particular link. There are a few limits, however, which affect these selections and the link feasibility. It is desirable to limit the peak transmitted energy density on the ground to values which are well below the accepted safety standards. Thus, for any given laser energy level, there is a minimum transmit beamwidth.

The optical receiver must employ relatively small active area detectors to obtain sufficient bandwidth for ranging purposes. Thus, there is a finite limit on the maximum receiver aperture which can be employed for any particular receiver field-of-view (FOV).

The first step in the link analysis process is to quantify these limits, and define a maximum performance system. Comparison of the achieved performance with the experiment objectives will then yield a performance margin, i.e., the amount by which the performance can be degraded and still achieve the experiment objectives. The final step in this process is to allocate the performance margin to system parameters in a maximally effective manner.

2.1.1 Maximum Performance System Analysis

The first limit to be considered is the minimum transmit beamwidth. This limit is imposed to assure that the maximum possible transmit energy density at the ground (E_d) does not exceed the safe level for human exposure. Neglecting system losses, and assuming a Gaussian intensity profile, it can be shown that,

$$\alpha_T \geq \sqrt{8E_p / \pi E_d h_s^2} \quad (1)$$

where

α_T = full planar beamwidth (e^{-2} power points)

E_p = laser energy/pulse (joules)

E_d = energy density limit (joules/m²)

h_s = spacecraft altitude (m)

The second limit to be considered is the maximum receiver aperture. This limit results from optical system considerations. The receiver FOV is simply the detector diameter (d) divided by the receiver focal length (f). The receiver optics F number is the focal length divided by the receiver aperture diameter (D_R). Thus, for a given optics system F number and detector diameter the maximum receiver aperture is,

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$$D_R \leq d/F \text{ (FOV)} \quad (2)$$

The minimum receiver field of view is nominally chosen to be equal to the transmit beamwidth (α_T). Thus, the maximum receiver aperture can be defined as,

$$D_R \leq \left(\frac{dh_s}{F} \right) \sqrt{\frac{\pi E_d}{8 E_p}} \quad (3)$$

The radar range equation which determines the return signal energy from the target can be written as,

$$N_S = E_P \eta G_T A_e \sigma S_T S_r S_a / (4\pi)^2 R^4 h \nu \quad (4)$$

where,

- N_S = signal p-e/pulse
- E_P = transmit energy/pulse (joules)
- G_T = transmit antenna gain
- $A_e = \pi D_R^2 / 4 \text{ (m}^2\text{)}$
- σ = target cross-section (m^2)
- S_T = transmitter optical efficiency
- S_a = two-way atmospheric transmissibility
- R = slant range (m)
- η = detector quantum efficiency
- S_r = receiver optical efficiency
- h = Planck's constant ($6.6256 \times 10^{-34} \text{ Js}$)
- ν = optical frequency ($5.66 \times 10^{14} \text{ Hz @ } 0.53\mu\text{m}$)

For a Gaussian beam, the transmit antenna gain is $32/\alpha_T^2$. Thus, substituting equations (1) and (3) into equation (4) yields,

$$N_{S_{MAX}} = \eta \left(\frac{h_s}{R} \right)^4 E_d^2 \left(\frac{d}{F} \right)^2 \pi \sigma S_T S_r S_a / 128 h \nu E_p \quad (5)$$

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The design range, R , is chosen as the range at which the elevation angle of the spacecraft, when viewed from the target, is E (nominally 20° minimum).

$$R = -r_e \sin E + \sqrt{(r_e + h_s)^2 - r_e^2 \cos^2 E} \quad (6)$$

where,

r_e = earth radius (m)

The next step is to determine the required signal energy to meet the ranging accuracy goal. Previously, we determined that the variance of the time of arrival estimation error, assuming a matched filter or correlator, was given by,

$$\overline{E_\alpha^2} = \frac{T^2}{2\pi} \left(\frac{N_S + N_B}{N_S} \right) \quad (7)$$

where,

N_S = signal photo-electrons/pulse

N_B = background photo-electrons/gate

A raised cosine pulse shape was assumed, and the pulse width at the half power points was $T/2$. The nominal gate width is T . In order to meet a goal of 10 cm

rms range measurement error, $\sqrt{\overline{E_\alpha^2}} = 0.667$ ns. Assume a pulsewidth (FWHM) of 5 ns. Then,

$$\frac{N_S^2}{N_S + N_B} = \left(\frac{2\pi^2 \overline{E_\alpha^2}}{T^2} \right)^{-1} = 11.4 \quad (8)$$

This equation describes the minimum signal to shot noise ratio (SNR) which will satisfy the 10 cm rms accuracy goal with a matched filter detector.

The effective background energy received is determined by,

$$N_B = T \left(\frac{\pi D_R^2}{4} \right) \Delta\lambda N \eta S_r \frac{\pi}{4} (\text{FOV})^2 / h\nu \quad (9)$$

where,

$\Delta\lambda$ = filter bandwidth ($\text{\AA}$)

N = background radiance ($\text{w/m}^2\text{-}\text{\AA}\text{-}\text{sr}$)

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Then, let $\text{FOV} = \alpha_T$, and substituting equations (1) and (3), we find,

$$N_B = T \Delta\lambda N \eta S_r \left(\frac{\pi d}{4F} \right)^2 / h\nu \quad (10)$$

At $0.532 \mu\text{m}$, the sunlit earth background radiance is approximately $0.017 \text{ w/m}^2\text{-A-ster}$. Assume $\Delta\lambda = 10 \text{ \AA}$, $T = 10 \text{ ns}$, $\eta = 0.25$, $S_r = 0.4$, $d = .005 \text{ m}$, $F = 2.5$.

$$N_B = 1118.53 \text{ p-e/gate}$$

Then, solving equation (8) we find the required N_S is,

$$N_S = 118.8 \text{ p-e/pulse}$$

Next, assume $h_g = 333 \text{ km}$ (180 nmi), $E = 20^\circ$, $S_T = S_a = 0.8$

Then, $R = 838 \text{ km}$, and,

$$N_{S_{\text{MAX}}} = 4.18 \times 10^8 E_d^2 \sigma / E_p \quad (11)$$

Next define the performance margin as the ratio,

$$M_p = (N_{S_{\text{MAX}}}) / (N_{S_{\text{RQD}}}) = 3.52 \times 10^6 E_d^2 \sigma / E_p$$

The maximum safe visible frequency energy density for human exposure is on the order of $10^{-7} \text{ joules/cm}^2$. In order to be conservative, choose $E_d = 10^{-5} \text{ joules/m}^2$ (20dB safety margin).

$$M_p = 3.52 \times 10^{-4} \sigma / E_p$$

Note that E_p appears in the denominator of this expression, implying that smaller energy per pulse lasers have a greater performance margin. This, of course, results from choosing the transmit beamwidth as small as possible and the receiver aperture as large as possible. In the experiment, we will choose E_p as large as practical. A value of $E_p = 50 \text{ mJ/pulse}$ @ $0.532 \mu\text{m}$ appears achievable, resulting in,

$$M_p = 0.00704 \sigma$$

The targets are assumed to be arrays of cube-corners with a maximum radar cross-section of 10^7 m^2 , resulting in $M_p \leq 7.04 \times 10^4$ (48.5 dB). A practical

system, therefore, can be configured which is up to 50 dB less efficient than the maximum performance system. Note that the transmit beamwidth and receiver diameter of the maximum performance system are 0.339 mrad and 5.9 m, respectively. A practical receiver diameter for a Shuttle flight is on the order of 15 cm (6 inches), or ~ 32 dB less receiver area than the maximum performance system. However, the received background energy is also reduced by 32 dB, which reduces the required signal energy by ~ 12 dB, for a net reduction of ~ 20 dB compared to the maximum performance system.

We conclude that a maximum performance system would collect approximately 50 dB more return signal than would be required to meet a 10 cm rms accuracy goal.

2.1.2 Multiple Target Ranging

The purpose of the laser ranging experiment is to obtain data which will allow determination of the relative distances between targets in a grid. The simplest approach, conceptually, is to emit a laser pulse which illuminates all of the targets in the grid, and measure the difference in the time of arrival of the reflections from each target. Then, after a first order correction for the range rate, the relative distances between the targets in the direction of the observation can be estimated with considerable accuracy. This process is then repeated for a number of observation angles to obtain a three dimensional representation of the target grid spacing.

This approach is reasonable where the physical size of the target grid is small, but quite difficult to implement where the grid size is large. Let D_g be the diameter of the minimum circle which just contains the target grid. Then, the minimum receiver FOV is simply D_g/h_g . The optimum transmit beamwidth is $(D_g/h_g) \sqrt{2}$.

As discussed in the preceding section, the maximum performance system had approximately 50 dB of margin above the 10 cm rms performance goal for a 50 mJ/pulse, 0.339 mrad system with a 5.9 m dia. collector. This would result in an illuminated spot on the earth of ~ 0.08 km diameter for a 333 km orbit altitude. If the transmit beamwidth and receiver FOV are increased to view a one km diameter grid, the performance loss is ~ 44 dB, or nearly all of the available 50 dB performance margin. In order to range from all targets in a

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10 km diameter grid, another 40 dB of performance loss could be encountered, requiring a total of 4 orders of magnitude increase in target cross-section and/or transmitter power. We concluded that this concept does not appear to be a feasible alternative. However, we elected to perform a more detailed parametric analysis to determine the limits for usable grid sizes.

One additional parameter, $\Delta\lambda$, the optical filter bandwidth, was included as a dependent variable. Although it is plausible to construct optical filters of nearly arbitrary bandwidths for narrow FOV applications, when the receiver FOV is on the order of a degree or more, existing filter technology limits the minimum usable bandwidth. Figure 1 shows one such projection, which is considered typical of the current state of the art. The smooth curve is the result of least square fitting the available data, and results in $\Delta\lambda \approx 3 + .155 \alpha_R^2$, where $\Delta\lambda$ is the filter bandwidth (Å) and α_R is the filter field of view (degrees). There is a current resurgence of interest in narrow bandwidth optical filters with wide usable cone angles. Techniques such as mosaics, selective gas absorption, and fiber optics channels are plausible. However, these concepts are not yet in the laboratory demonstration phase, thus it was judged inappropriate to demand better than currently available optical filter performance. We assumed, then, that the filter bandwidth followed the square law expression, above, and that the filter was located, in effect, at the receiver aperture. Note that the effective filter field of view is therefore equal to the receiver field of view; if a smaller diameter filter were used, elsewhere in the optics train, the filter FOV would be greater than the receiver FOV.

Typical high speed photo-multipliers, such as a static crossfield PMT, have a small (4-6 mm) photo-cathode submerged inside the tube, with a modest sized window (~.5" diameter). Typically, these detectors require the optics system to have F numbers on the order of 2.5 or more.

If the tube were redesigned, it is possible to hypothesize somewhat larger photo cathodes with a lower allowable F number, but order of magnitude improvements are considered virtually unattainable in a high speed device. For feasibility evaluation purposes, we chose to assume a detector diameter of 10 mm, and a minimum F number of 1.2. These limits are used at 0.53 μ m wavelength. However, at 1.06 μ m, the situation is somewhat different, since the most attractive detectors are photo-diodes, with or without avalanche gain. These devices

FILTER PERFORMANCE DATA

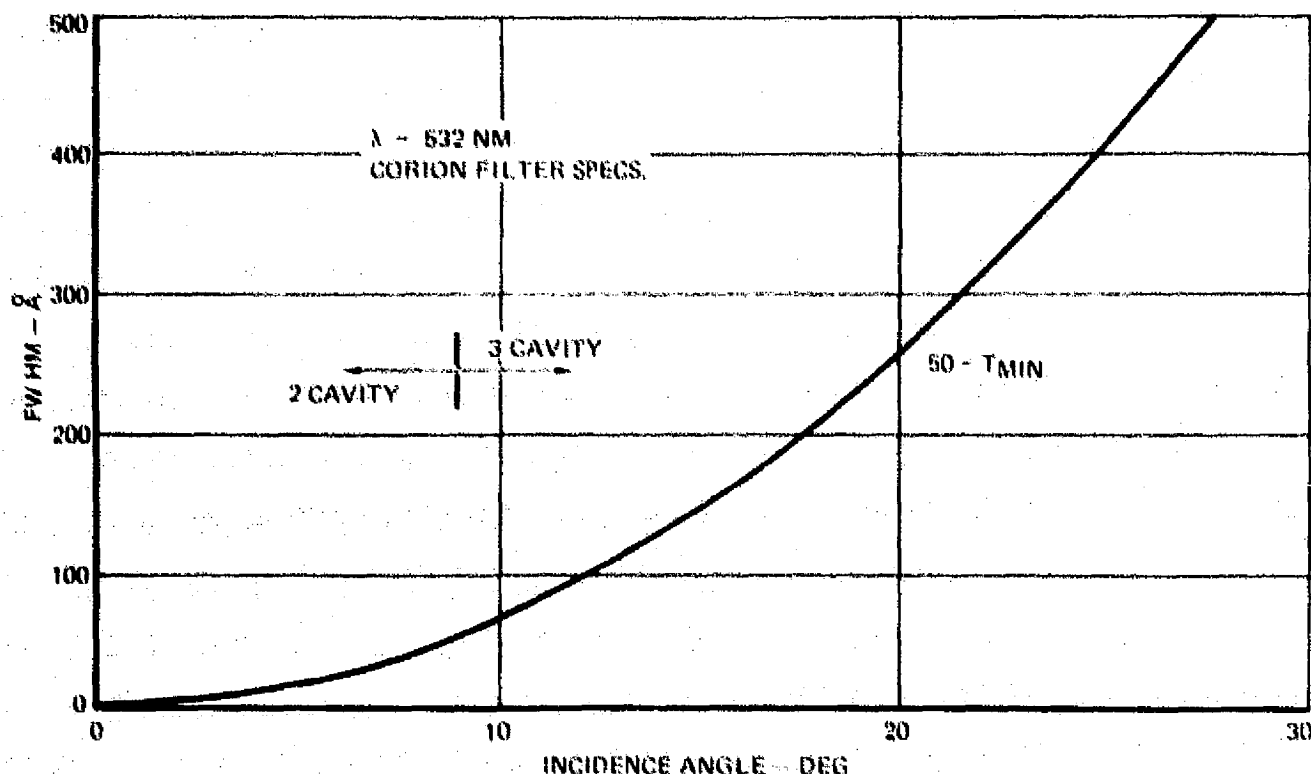


Figure 1

are limited to sizes on the order of 1 mm diameter (active area) in order to obtain sufficient bandwidth. However, it is possible to use special techniques to decrease the allowable F number to ≈ 0.5 .

Finally, we elected to determine the required signal energy for an optimal time of arrival estimator rather than the matched filter used in the previous section. For a maximum likelihood (ML) time-of-arrival estimator (with a raised cosine pulse), the variance of the estimation error is,

$$\overline{\epsilon_a^2} = \frac{T^2}{4\pi^2 \left[N_S + N_B - \sqrt{N_B^2 + 2 N_S N_B} \right]} \quad (12)$$

This can be inverted to obtain,

$$N_S = K + \sqrt{2 K N_B}, \text{ where } K = \left(\frac{T}{2\pi} \right)^2 / \overline{\epsilon_a^2} \quad (13)$$

The required signal energy for a ML estimator is only slightly less than for a correlator when the background level is large. The ML advantage increases as

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the background level is decreased, and approaches 3 dB at zero background level. For feasibility evaluation purposes, we would like to determine the performance parametrically with respect to the target grid dimensions. From a practical standpoint, the transmit energy per pulse is limited to about 50 mJ, hence, the only unconstrained variable remaining is the target cross-section. Thus, we elected to solve for the required target cross-section as a function of grid diameter.

The reference system is based on the guidelines in the SOW, i.e., 50 mJ per pulse @ 0.53 μ m wavelength, with a 5 ns (FWHM) pulse width. We assumed the receiver aperture was no greater than 0.15 m (6 in.), and a 25% quantum efficiency detector would be used. The transmit and receive optics were assumed to have 80% and 40% transmissibility, respectively. Atmospheric transmissibility is generally expressed as $\exp\{-K/\sin E\}$, where E is the elevation angle of the propagation path, measured at the target, and K is a constant on the order of .08 (one-way) at 0.53 μ m wavelength on a clear day.

Figure 2 shows the results for the nominal system, at spacecraft altitudes from 200 km to 400 km in 50 km steps, for 20° minimum elevation angle, assuming sunlit Earth background. A reasonable maximum target cross-section is probably on the order of $10^{7.2} \text{ m}^2$, thus we see that it is virtually impossible to obtain satisfactory operation at grid sizes greater than 1 km. Note that these results are for zero-margin with an ideal detector (ML). If we allow 6 dB margin for real detection processes, the design point would be for $\sigma = 2.5 \times 10^{6.2} \text{ m}^2$, which limits the grid to sizes between 0.4 km and 0.6 km.

Figure 3 shows the results, for the same conditions, for the 1.06 μ m variant of the reference system. As usually found, the performance at 1.06 μ m is worse than at 0.53 μ m, thus little reason can be found to pursue this alternative. The major problem with the broad beam approach, of course, is the reduction of energy density at the Earth surface as the transmit beamwidth is increased. Thus, we hypothesized a multiple transmit beam, single receiver system concept, which could, effectively, accomplish the multitarget ranging objective. Figure 4 shows the results under the 20° minimum elevation angle constraint. Although the $10^{7.2} \text{ m}^2$ (zero margin) performance has been extended to 14 to 24 km, significant improvement does not appear plausible. If the minimum required elevation angle is increased to 30°, operation with grid sizes up to ~30 km appears feasible, as seen in Figure 5, but larger grids appear unreachable.

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TARGET CROSS-SECTION PARAMETERS

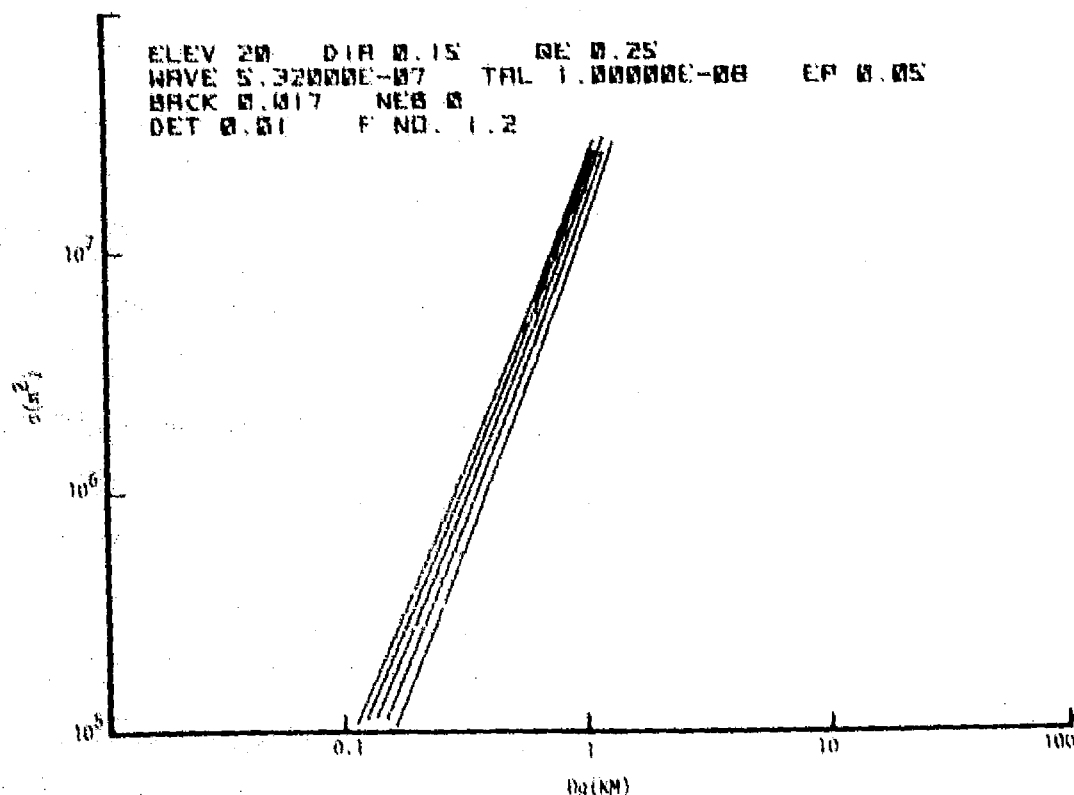


Figure 2

TARGET CROSS-SECTION PARAMETERS

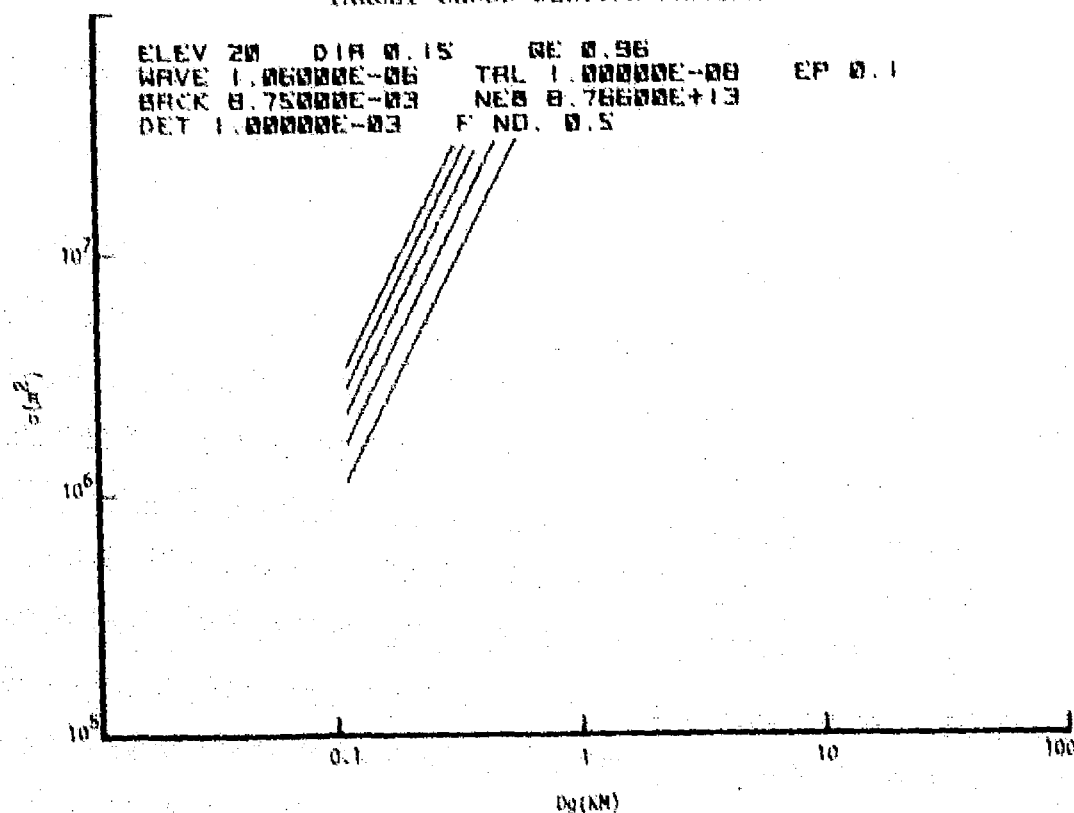


Figure 3

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TARGET CROSS-SECTION PARAMETERS

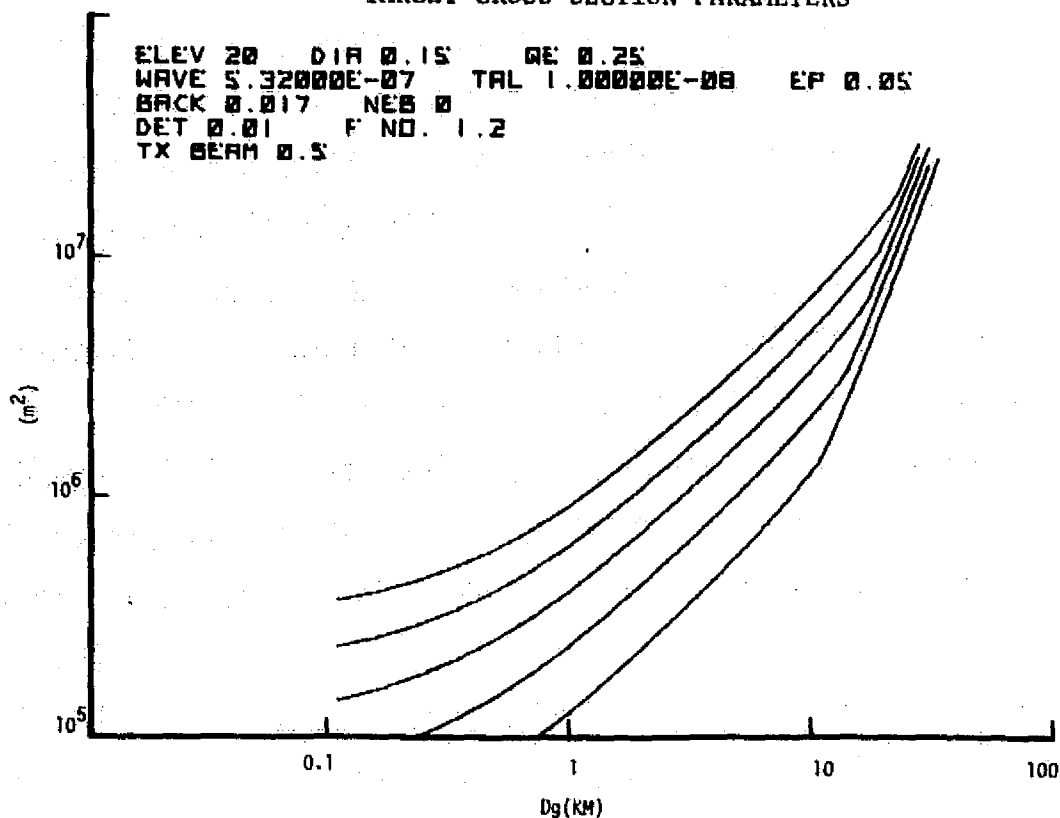


Figure 4

TARGET CROSS-SECTION PARAMETERS

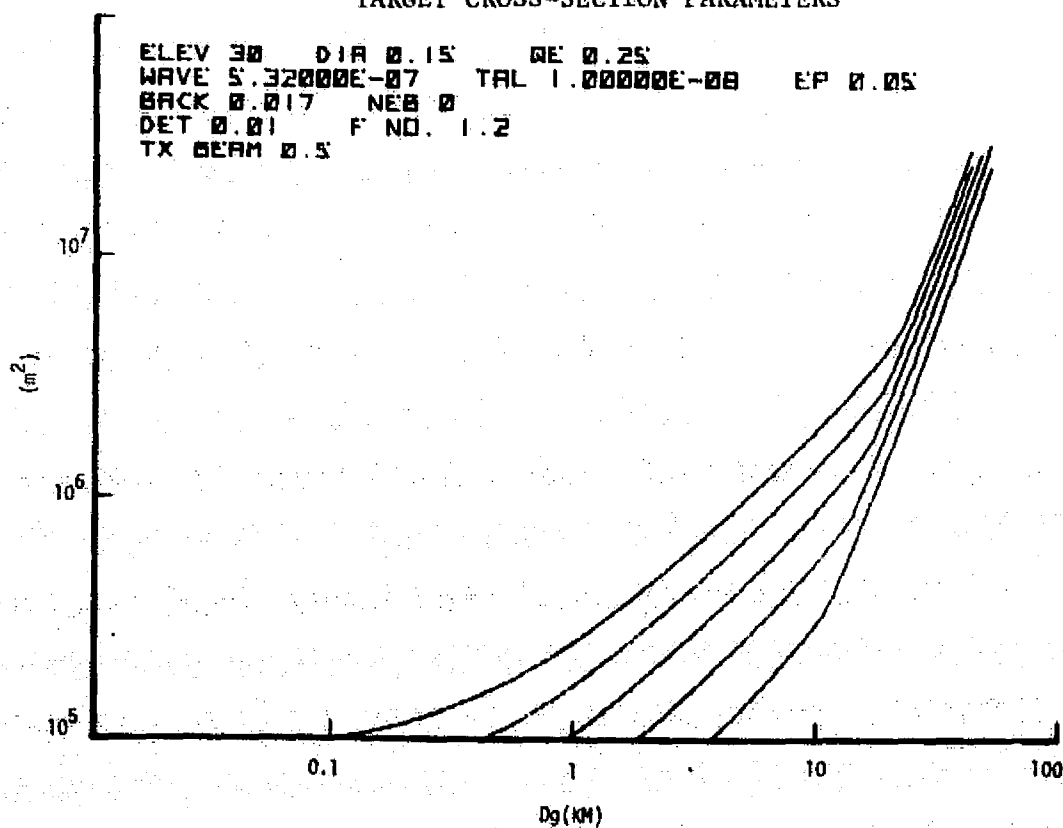


Figure 5

One possible alternative is to design the system for operation only at night. Although operationally undesirable, the improvement in performance capabilities may justify the decision. Figure 6 shows the results for the wide beam concept, with the background radiance set at zero. Clearly, the broad beam concept is simply not a viable alternative for grid sizes greater than 1 km, even with zero background. Figure 7 shows the results for the narrow beam wide FOV concept. The influence of the detector size on the receive aperture is clearly seen since the required target cross-section is constant until increasing FOV requirements begin to limit the maximum receive aperture size.

The concept of zero background level, however, is never really encountered. Thus, before we conclude that the concept is feasible, we must make an assessment of the impact of some modest background level on the performance. Since the Moon is the brightest object in the night time sky, we can estimate the background radiance of the Earth at night due to moonlight as the ratio

TARGET CROSS-SECTION PARAMETERS

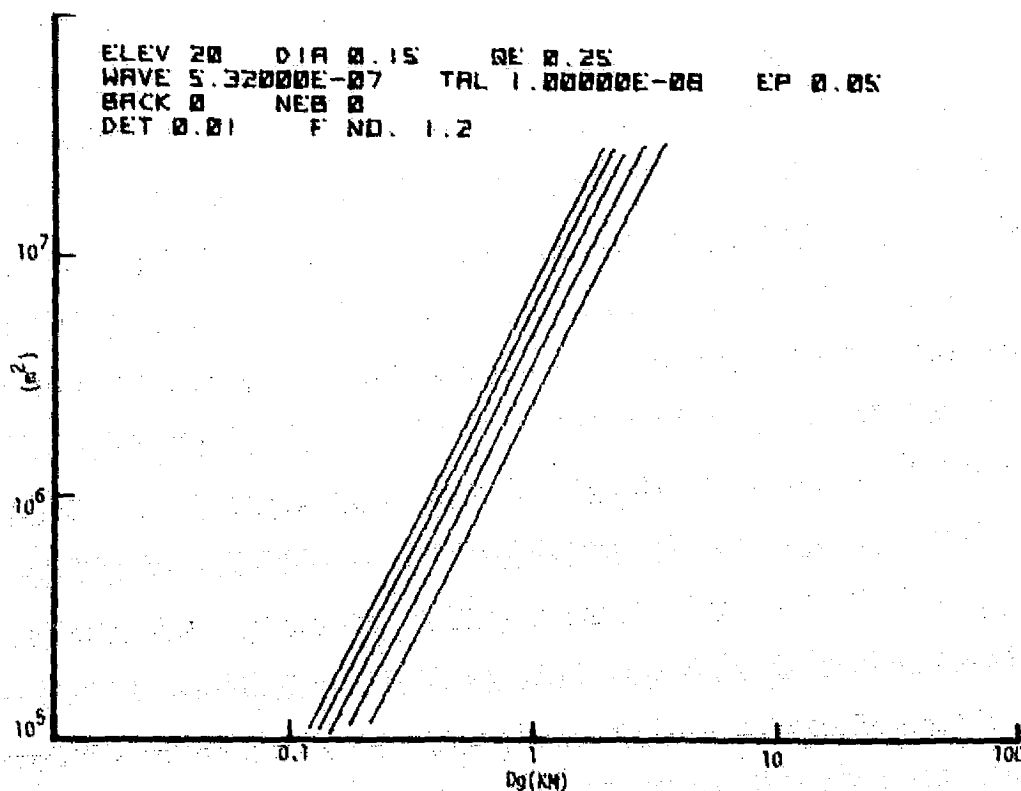


Figure 6

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TARGET CROSS-SECTION PARAMETERS

ELEV 20 DIA 0.15 DE 0.25
WAVE 5.32000E-07 TAL 1.00000E-08 CP 0.05
BACK 0 NEB 0
DET 0.01 F NO. 1.2
TX BEAM 0.5

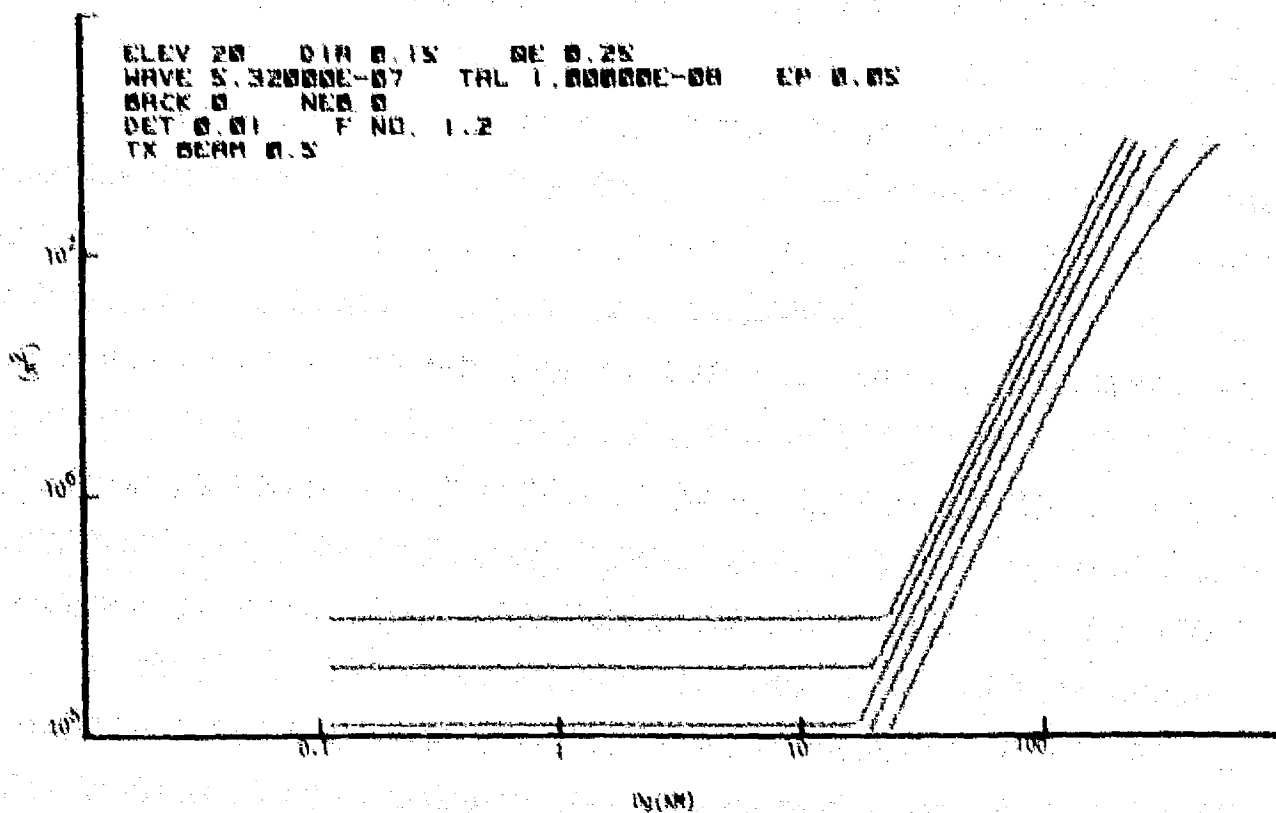


Figure 7

of lunar-to-solar spectral radiance times the radiance of the sunlit Earth, i.e., $(4.7 \times 10^{-3} / 2820) \times .017 \approx 2.8 \times 10^{-8} \text{ w/m}^2 \text{ -A-}\Omega$.

In order to be conservative, we increased the background level three orders of magnitude, with the results shown in Figure 8. We conclude that neither the wide beam nor the narrow beam, wide FOV concept is viable for modest to large size grids.

2.1.3 Time-of-Arrival Estimation

Three time-of-arrival (TOA) estimation techniques were investigated for the laser ranging experiment, a matched filter technique and two sliding window integrator techniques. The relevant performance analyses are discussed in the next paragraphs.

Sliding Window Integrator

A sliding window integrator can be constructed, conceptually, as shown in Figure 9. This approach uses a threshold detector to determine the TOA

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TARGET CROSS-SECTION PARAMETERS

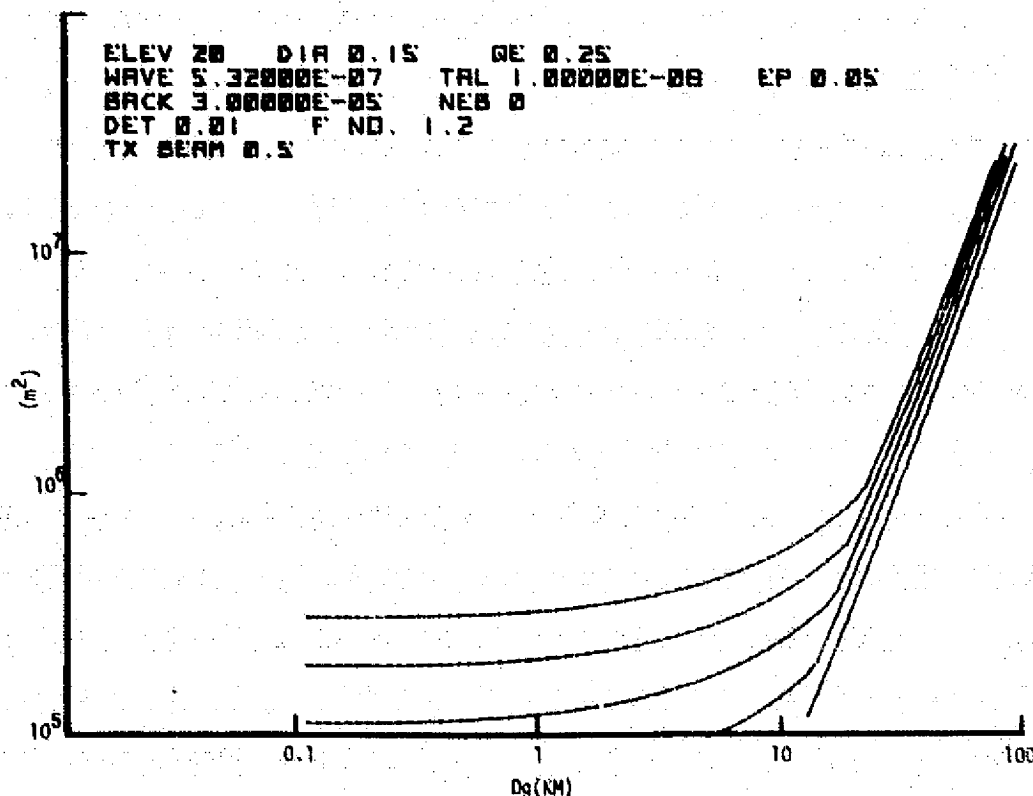


Figure 8

estimate. An automatic threshold controller (ATC) adjusts the detection threshold to minimize false alarms. The TOA estimate, for this approach, is not ideal since it tends to be biased if the mean signal level is either greater than or less than approximately twice the threshold setting. The concept is useful, however, for analyzing false alarm and missed detection statistics, particularly where the range gate is large (compared to the pulsewidth) during acquisition.

SLIDING WINDOW INTEGRATOR BLOCK DIAGRAM

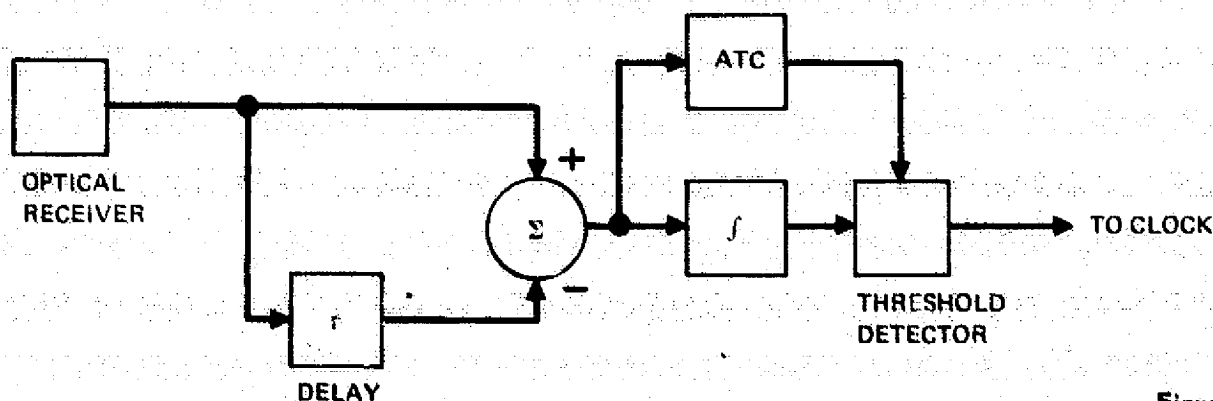


Figure 9

During the acquisition mode, the laser ranging system will be searching in angular space and time to locate the target. If a false alarm occurs, an unnecessary target verification sequence will be executed, resulting in a delay of correct target acquisition (after the search is resumed). Thus, a single false alarm is simply a nuisance. Only if false alarms are frequent will any serious problems be encountered. The nominal acquisition period is on the order of 2 to 20 seconds; at ten pulses/second, there are a maximum of 200 opportunities for false alarms. A single trial false alarm probability of 1/2000 will therefore limit the probability of more than one false alarm during acquisition to about 1%.

Previously, we found that the probability of false alarm in a single trial for a sliding window integrator could be expressed as,

$$P_{FA} = 1 - e^{-a_L T} \quad (14)$$

where T = scan time (sec)

$$a_L = \frac{n_b (n_b \tau)^{L-1}}{(L-1)!} \left[\sum_{K=0}^{L-1} \frac{(n_b \tau)^K}{K!} \right]^{-1}$$

and

n_b = mean background rate (p - e/sec)

τ = gate width (sec)

L = threshold level (p - e)

when $L \gg n_b$, and $a_L T \ll 1$,

$$P_{FA} \approx n_b T \frac{(n_b \tau)^{L-1}}{(L-1)!} e^{-n_b \tau} \quad (15)$$

Clearly, for any particular background rate, gate width, and scan time there is a threshold setting which will limit the false alarm probability to an acceptable value. Note, however, that for a spacecraft in low altitude orbit, the mean background rate observed with a narrow field of view detector can vary rather rapidly. We devised an automatic threshold control (ATC) technique to cope with a rapidly varying background level.

Assume that the signal, if present, will be in a scan time window, T_1 . During a time period, T_2 , immediately preceding the scan time window, the output of the sliding window integrator is fed to a peak detector. The threshold is then set just above the maximum integrator output sensed in the T_2 seconds time period. The threshold setting, L_1 thus selected is a random variable. The probability of selecting a specific value, L_1 , is,

$$p(L) = e^{-a_L T_2} - e^{-a_{L-1} T_2} \quad (16)$$

Then, we see that the probability of a false alarm during the scan time window, assuming n_b is constant during the $T_1 + T_2$ time period, is simply,

$$\text{Pr}\{\text{FA}\} = \sum_{L=1}^{\infty} (1 - e^{-a_L T_1}) (e^{-a_L T_2} - e^{-a_{L-1} T_2}) \quad (17)$$

This equation has been evaluated numerically. We assumed $T_1 = 10\tau$, and varied T_2 from 100τ to $10^5\tau$ for a widerange of background levels ($n_b\tau = .001$ to 100); the results are shown in Figure 10. As can be seen, $T_2 = 10^4\tau$ to $10^5\tau$ results in a usable false alarm probability ($<10^{-3}$) over the range of background levels examined. The behavior of the false alarm probability curves for small background levels reflect the transitions of the pump-up period as larger threshold settings become probable. These transitions become blurred as the background level increases.

The time available for the pump-up period is the time between departure of the transmit pulse and the beginning of the range gate, which reaches a minimum of ~ 2.2 ms for a direct overhead pass. If we allow ~ 1 ms for the pump-up period, and assume $\tau = 10$ ns, $T_2/\tau = 10^5$, and we can expect satisfactory results for a 100 ns range gate. During initial acquisition, however, the range gate is considerably larger, more like 3 μ s. Figure 11 shows the performance of the ATC for $T_1 = 300\tau$. Note that for $T_2 = 10^5\tau$, the false alarm rate for $n_b\tau = 10$ has increased from 6×10^{-5} to 1.8×10^{-4} , a factor of 30 increase, showing a nearly linear dependence of P_{FA} to both T_1 and T_2 where P_{FA} is small and $n_b\tau \geq 1$.

Thus, we conclude that a pump-up ATC with a sliding window integrator is a satisfactory technique to control the false alarm probability over wide range

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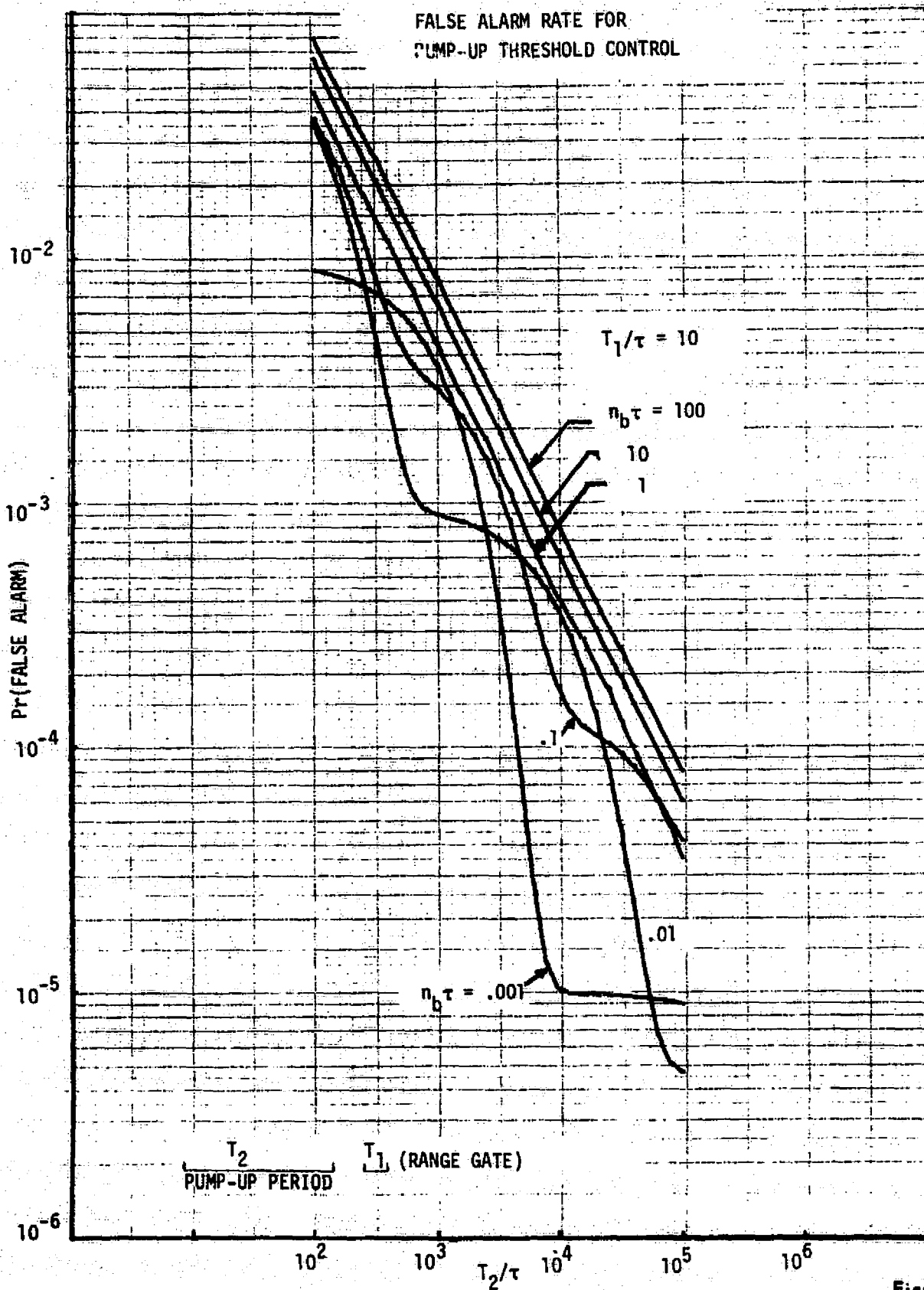


Figure 10

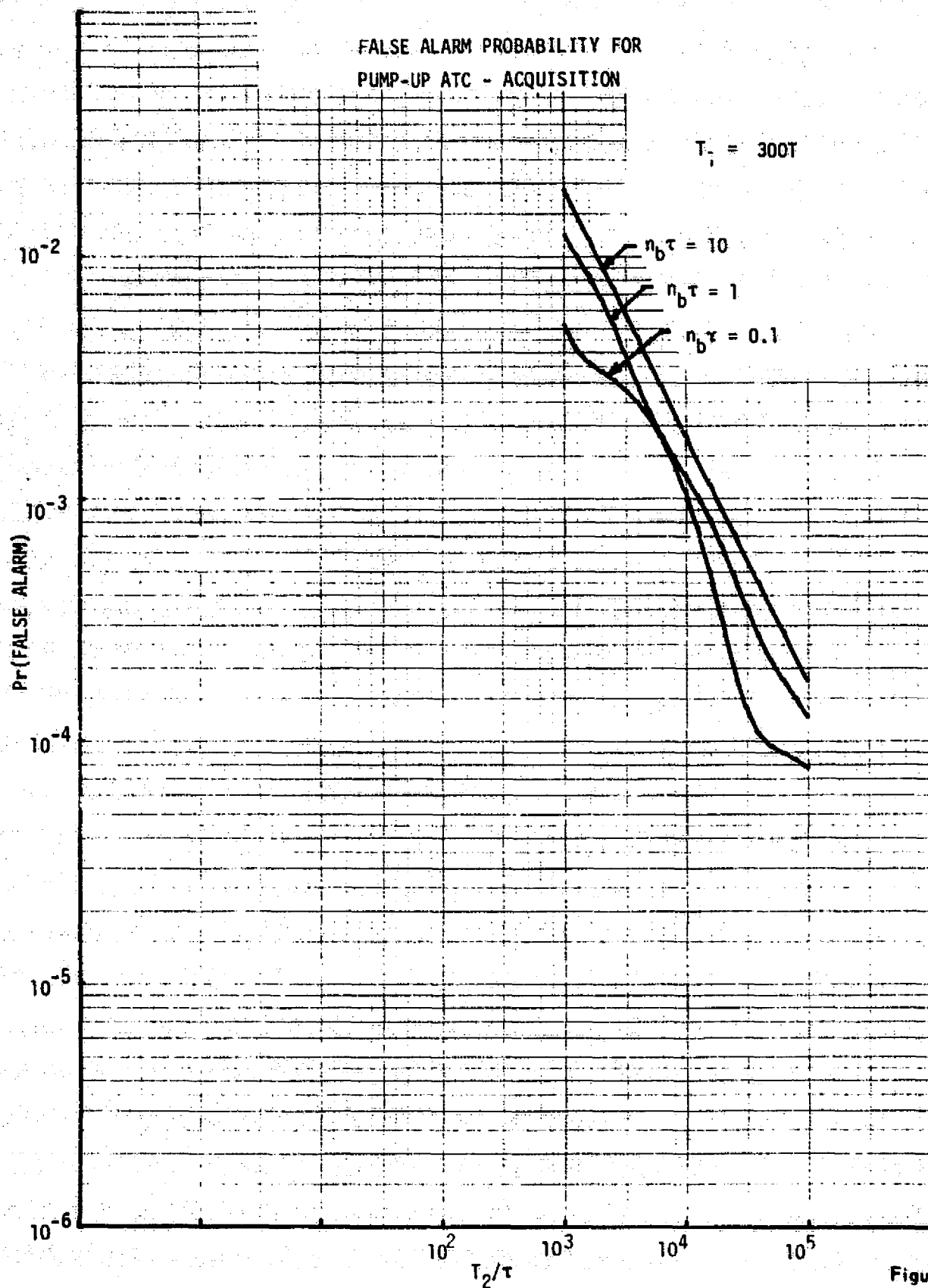


Figure 11

of background levels. Further, a pump-up time period of 1 ms or less will nominally result in a satisfactory false alarm probability, and can be accommodated readily.

The next step is to compute detection probabilities for the pump-up ATC system when the return pulse is present. This can be estimated very easily by simply considering the detection probability where the signal pulse is contained wholly within the sliding window integrator. If we ignore false alarms

$$P_{MD} = \sum_{L=1}^{\infty} (e^{-a_L T_2} - e^{-a_{L-1} T_2}) \sum_{K=0}^{L-1} \frac{(n_b \tau + N_s)^K}{K!} e^{-(n_b \tau + N_s)} \quad (18)$$

This equation has been evaluated numerically with the results shown in Figures 12 and 13. Comparison shows that increasing T_2 has a small effect on the required signal energy for a given missed detection probability. Figure 14 shows the missed detection probability for a stationary threshold system for comparison purposes. The threshold was selected to keep $P_{FA} \leq 10^{-4}$, and $T_1 = 10\tau$. As can be seen, the difference between the curves for the random threshold with $T_2 = 10^4 \tau$ and the stationary threshold is very small (< 2 signal p-c). Thus, we conclude that the pump-up ATC system concept is a viable candidate technique for the laser ranging system.

Correlator Performance

Previously, we determined that a correlator or matched filter technique yielded improved time-of-arrival estimates compared to a sliding window integrator with threshold detection. One possible implementation technique is to employ a tapped delay line with pulse shape weighted summing of the outputs.

Figure 15 shows a typical example, with six taps. Since a finite number of taps are to be used, it is desirable to determine the degradation in performance as compared to a continuous matched filter. Consider the case where the pulse shape is well approximated as a raised cosine.

$$\lambda(t) = n_s (1 + \cos 2\pi t/T), \quad |t| \leq T/2 \quad (19)$$

$n_s T$ is the mean energy in the received pulse.

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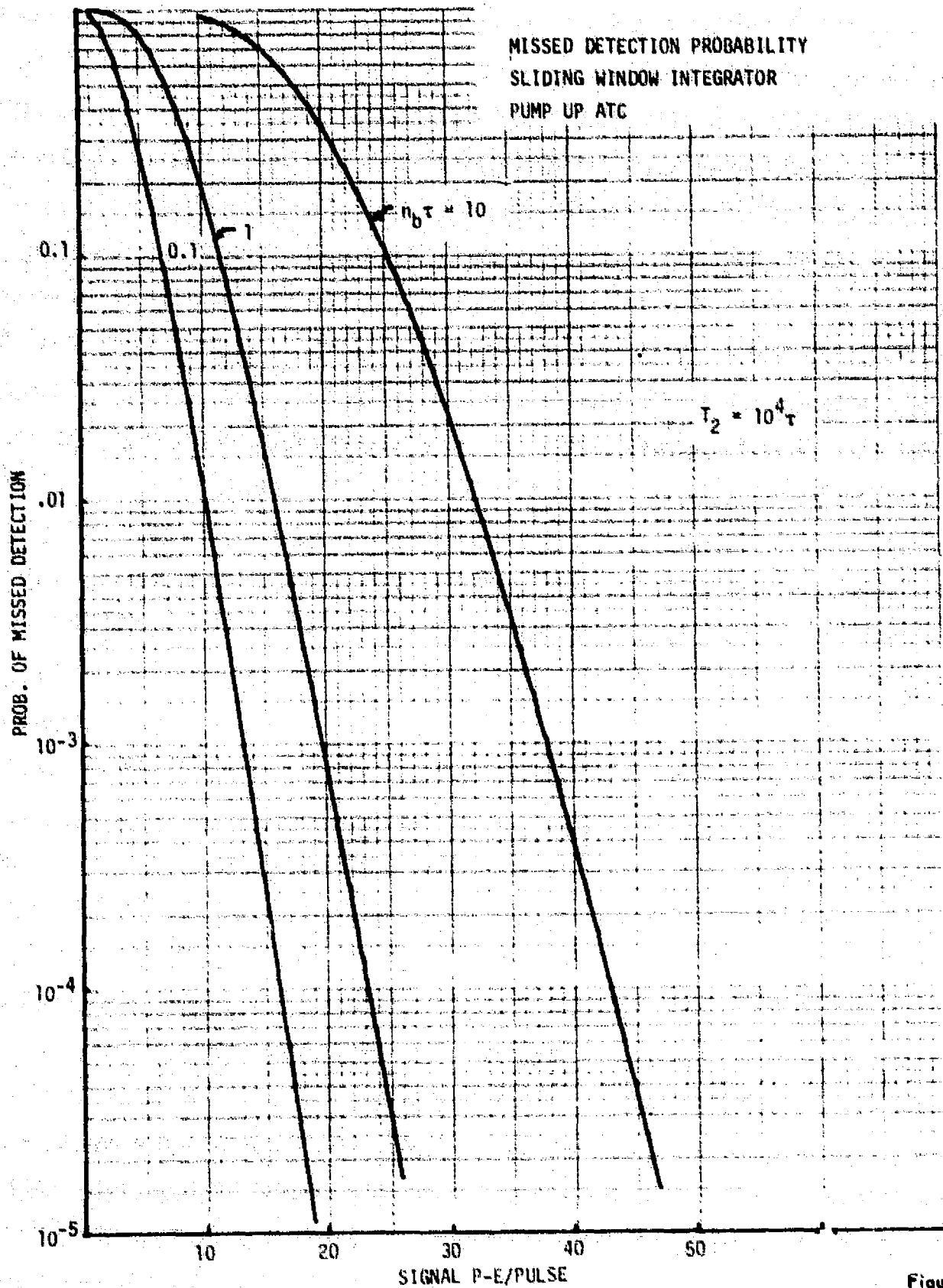


Figure 12

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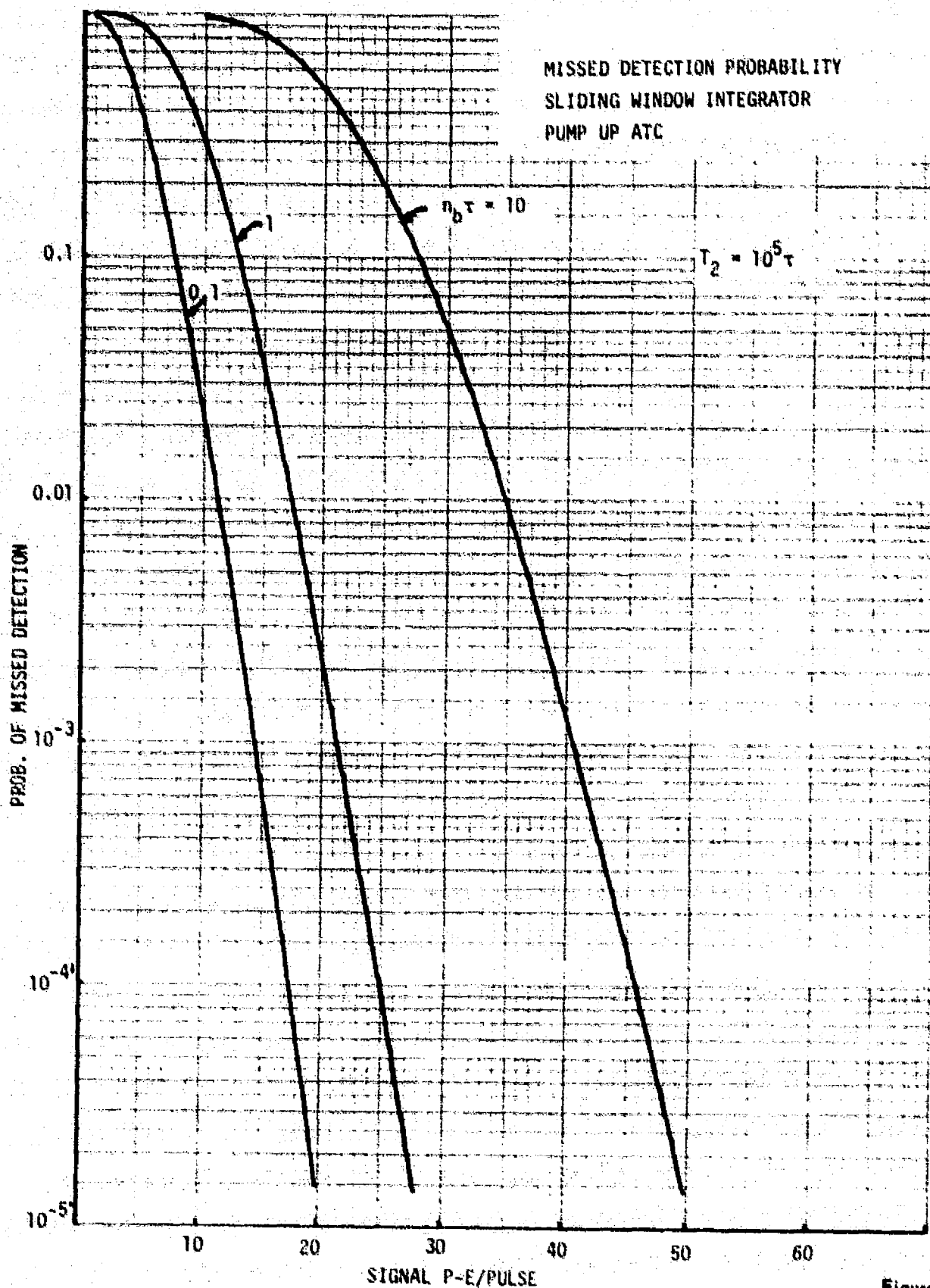


Figure 13

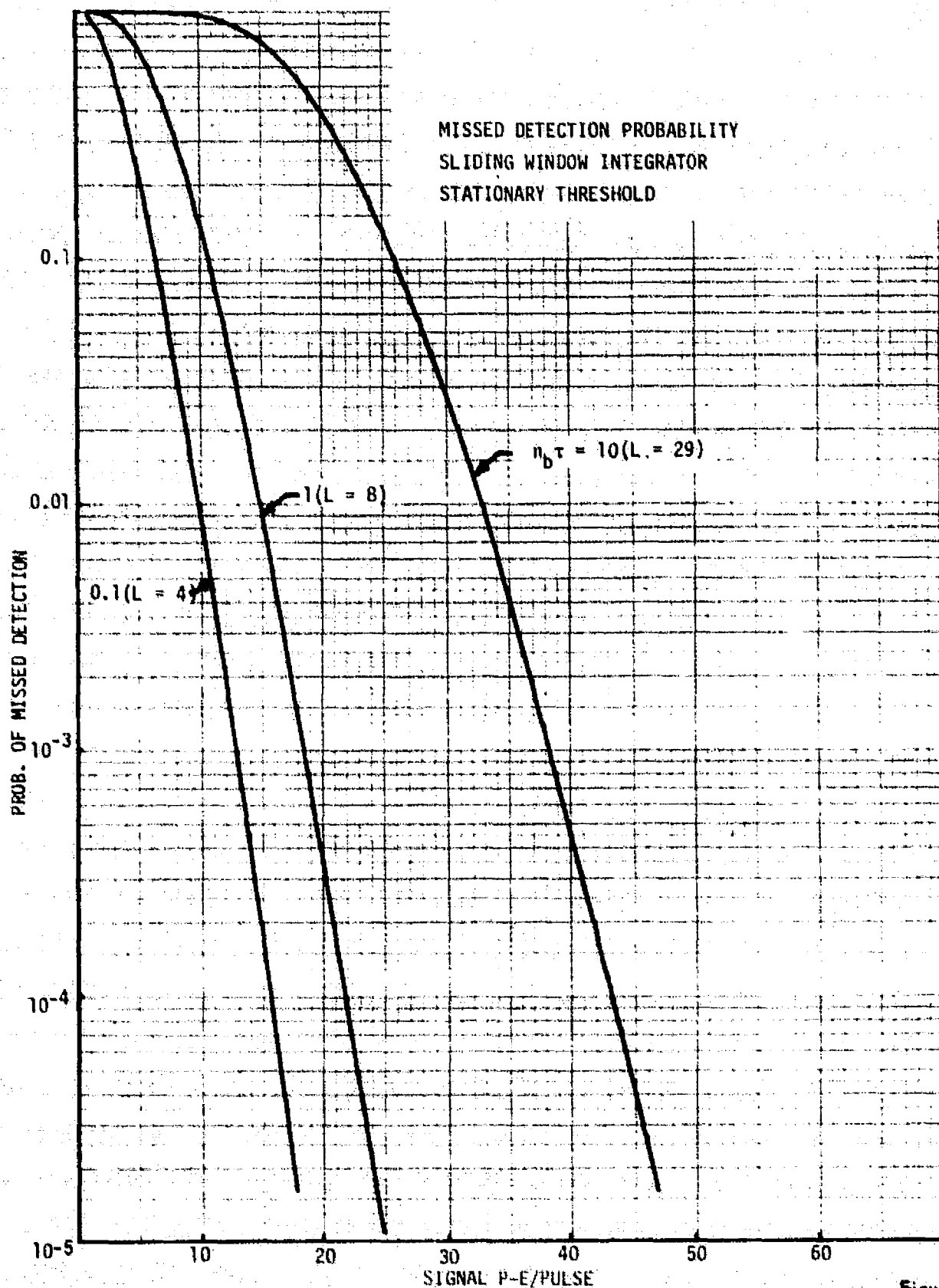


Figure 14

CORRELATOR SCHEMATIC

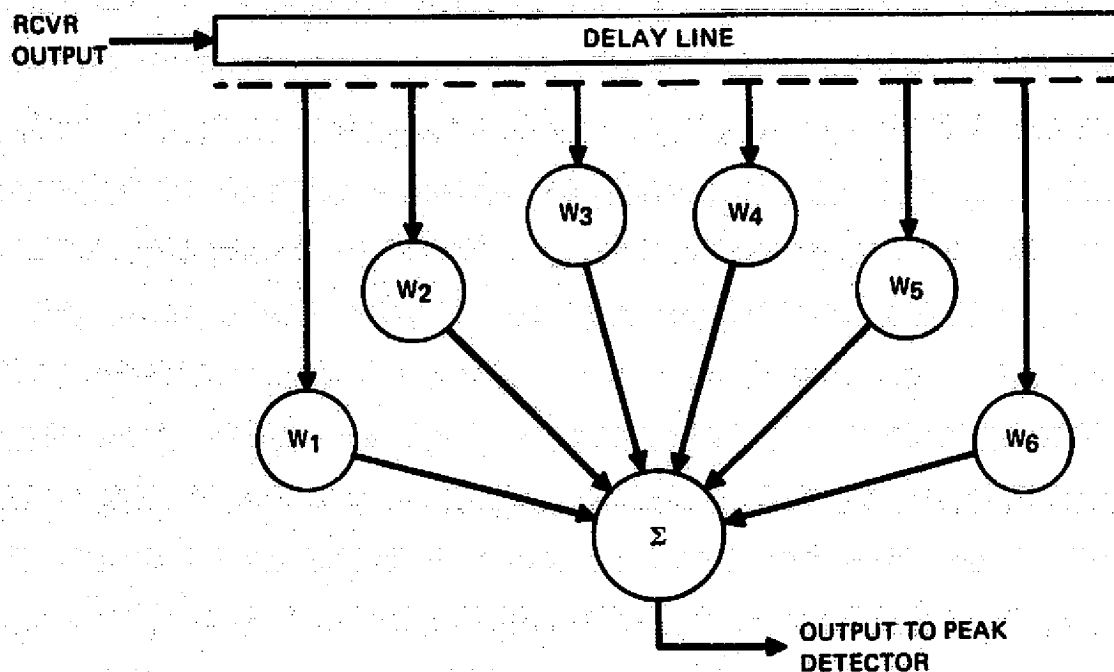


Figure 15

When the received pulse is exactly centered in the delay line, the mean signal energy in the j th bin is simply

$$N_{s,j} = n_s \left[T/N + \frac{T}{\pi} \sin(\pi/N) \cos \left(2\pi \left(\frac{j-0.5}{N} - 1/2 \right) \right) \right] \quad (20)$$

where N = number of taps.
Assume the weighing functions are,

$$W_j = 1 + b \cos \left(2\pi \left(\frac{j-0.5}{N} - 1/2 \right) \right) \quad (21)$$

where b is a constant used to vary the filter characteristics.

The second moment generating function of the output of the j th tap is,

$$\psi_j(S) = (N_{s,j} + n_b T/N) (e^{W_j S} - 1) \quad (22)$$

The second moment generating function of the output of the summer is,

$$\psi_x(S) = \sum_{j=1}^N \psi_j(S) = \sum_{j=1}^N N_{s,j} e^{W_j S} + \frac{n_b T}{N} \sum_{j=1}^N e^{W_j S} - \sum_{j=1}^N N_{s,j} - n_b T \quad (23)$$

where n_b is the background rate in p-e/sec.

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In order to reduce this expression to a more compact form, we will use the following relationships.

$$e^{Z \cos \theta} = I_0(Z) + 2 \sum_{K=1}^{\infty} I_K(Z) \cos k\theta \quad (24)$$

$$\sum_{j=1}^N \sin jx = \sin(Nx/2) \sin((N+1)x/2) / \sin x/2 \quad (25)$$

$$\sum_{j=1}^N \cos jx = \sin(Nx/2) \cos((N+1)x/2) / \sin x/2 \quad (26)$$

We first expand $e^{W_j S}$ as indicated in the first relationship, and then perform the trigonometric summations, and find,

$$\psi_x(S) = n_b T [I_0(bs) e^S - 1] + n_s T [I_0(bs) + I_1(bs) \frac{N}{\pi} \sin \pi/N e^S - 1] \quad (27)$$

Then, noting that $\psi_x^1(0) = n_x$ and $\psi_x^{11}(0) = \sigma_x^2$, we see,

$$n_x = n_b T + n_s T (1 + \frac{b}{2} \frac{N}{\pi} \sin \frac{\pi}{N}) \quad (28)$$

$$\sigma_x^2 = n_b T (1 + b^2/2) + n_s T (1 + b^2/2 + b \frac{N}{\pi} \sin \frac{\pi}{N}) \quad (29)$$

Thus, we see that with $b = 0$, the filter is a simple integrator, and the mean and variance are equal, as expected for an ideal photoselectron counter. Where $b = 1$, and N is large,

$$n_x \approx n_b T + \frac{3}{2} n_s T \quad (30)$$

$$\sigma_x^2 \approx \frac{3}{2} n_b T + \frac{5}{2} n_s T \quad (31)$$

The next step is to estimate the effect of varying N and b on the performance of the system. One of the more significant criteria is detection probability and false alarm probability vs threshold setting.

In order to estimate these probabilities, we use Chernov bounds as given below.

$$\Pr \{X < \gamma\} \leq \exp [\psi_x(s) - S\psi_x'(s)] / \gamma = \psi_x'(s), \quad S < 0 \quad (32)$$

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$$\Pr \{X > \gamma\} \leq \exp [\psi_x(s) - S\psi_x(S) - S\psi_x'(S)]/\gamma = \psi_x'(s), S > 0 \quad (33)$$

We evaluated these bounds numerically, with the results shown in Figures 16 and 17. Although varying b causes both the false alarm and missed detection curves to shift, the effect on detection probability is rather small, as can be seen from the intersections of these curves (highlighted with a circle). We could conclude that there is an optimum b (a function of background level), but the effect is small enough that optimizing b is not a high payoff activity.

Varying the number of taps does not affect the false alarm statistics, and has only a small effect on the missed detection probability as can be seen from Figure 17.

We were concerned, however, that the finite number of taps might cause the output of the summer to be somewhat distorted. This was evaluated with a simulation, with the results shown in Figures 18 through 20. The delay line was simulated with 40 discrete time slots, and the effect of various groupings tested with a repeatable random sequence. In the first figure, each time slot was weighed individually, and the resultant curve of the summer output is quite smooth. The next step was to group the time slots into 8 taps (five time slots each), with a resulting summer output curve shown in Figure 19. Some distortion is evident, but of little significance. Figure 20 shows that for 4 taps, a significant distortion is observed, which could cause problems in determining the peak. Thus, we concluded that 8 taps is a reasonable choice. Four additional runs were made in the 8 tap configuration using different random sequences, with the results shown in Figure 21. Although some roughness is apparent, the pulses are reasonably smooth and the peaks detectable.

Alternatively, the correlator could be implemented slightly differently such that a zero-crossing detector could be used to detect the pulse center. This would be accomplished by changing W_j to be,

$$W_j = \cos (2\pi ((j + .5)/N + 0.25)) \quad (34)$$

Figure 22 shows the summer output as determined by simulation for three pulses. The negative transient preceding the zero-crossing would trip a threshold detector used to arm the zero-crossing detector. Determination of the performance of

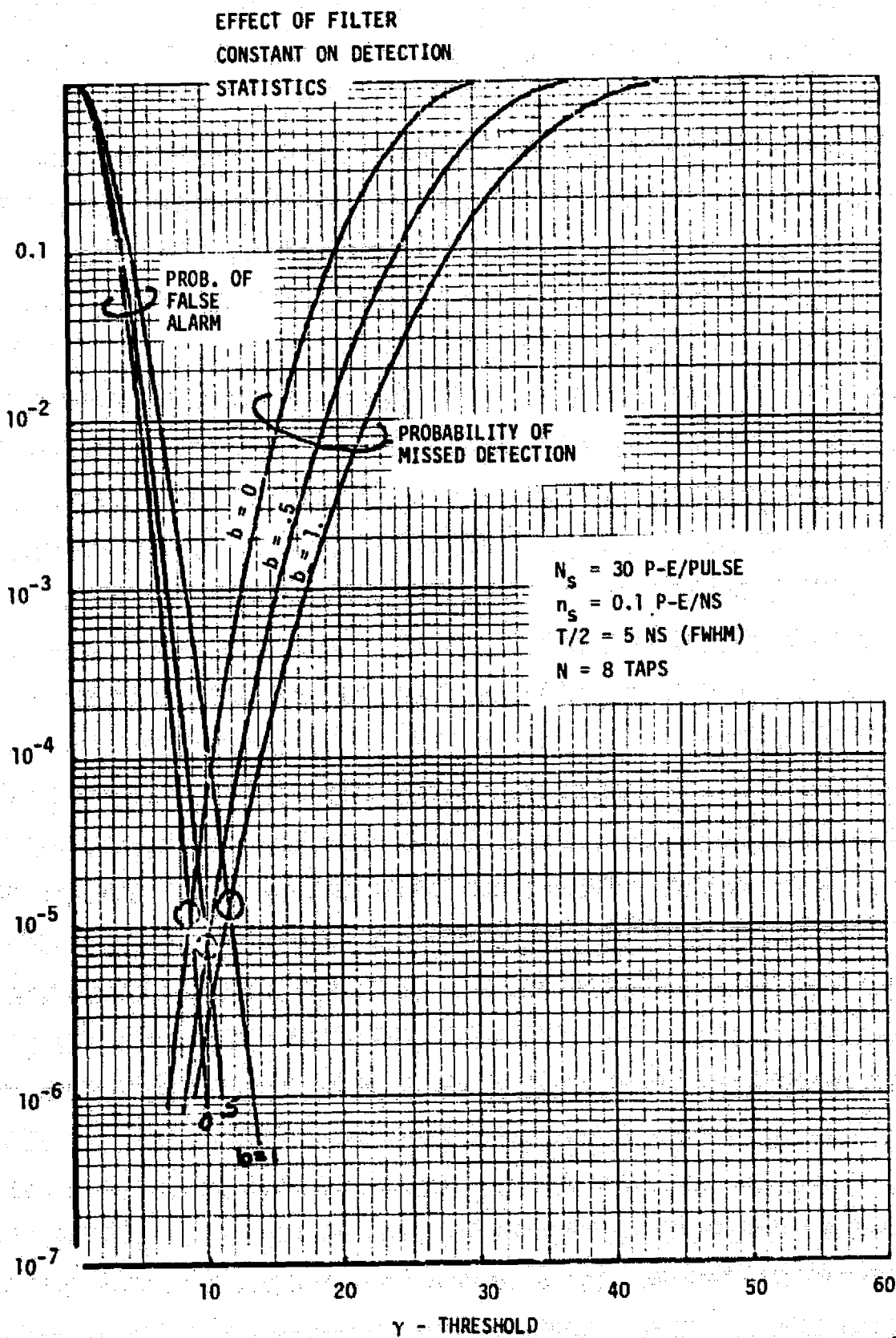


Figure 16

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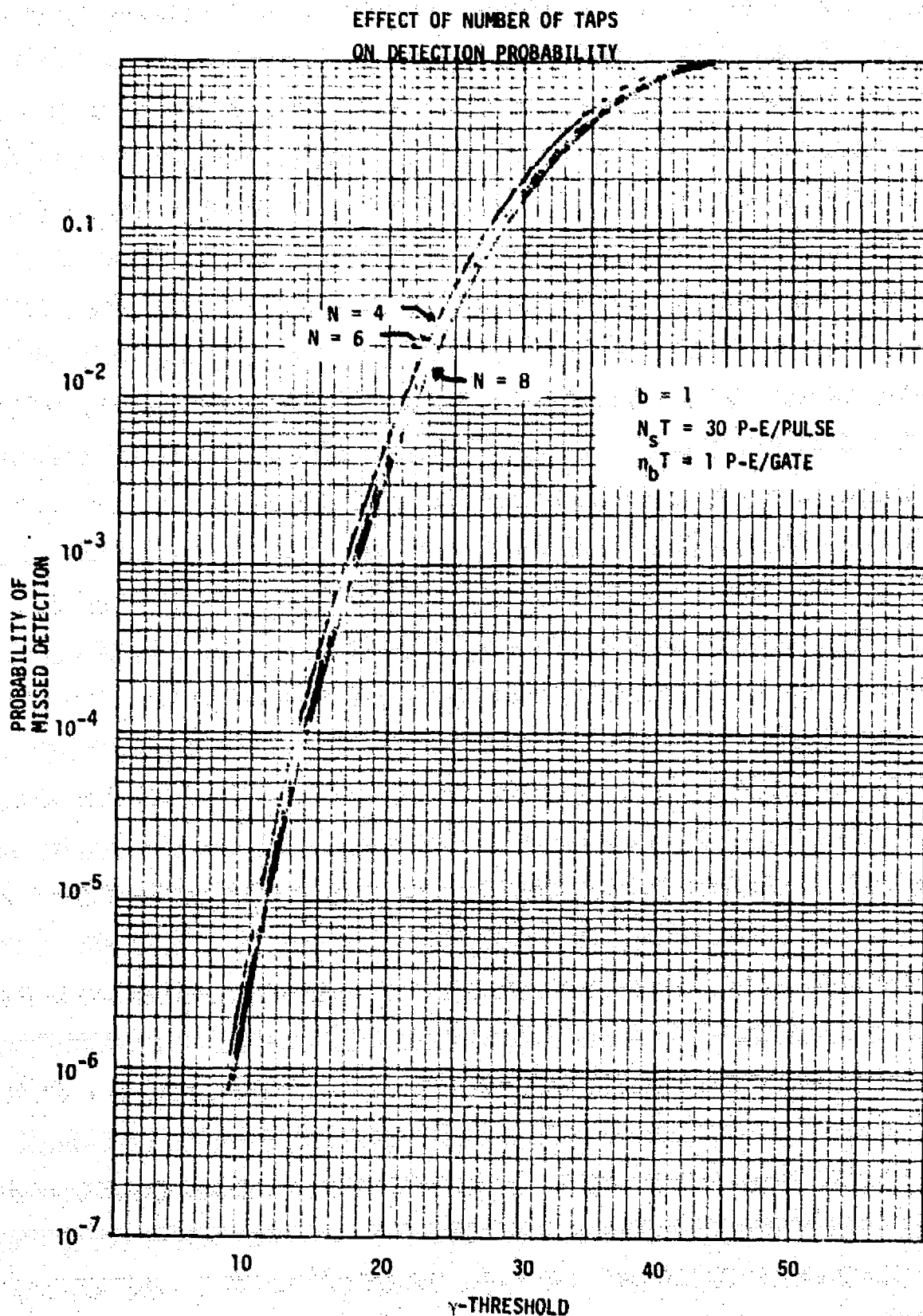
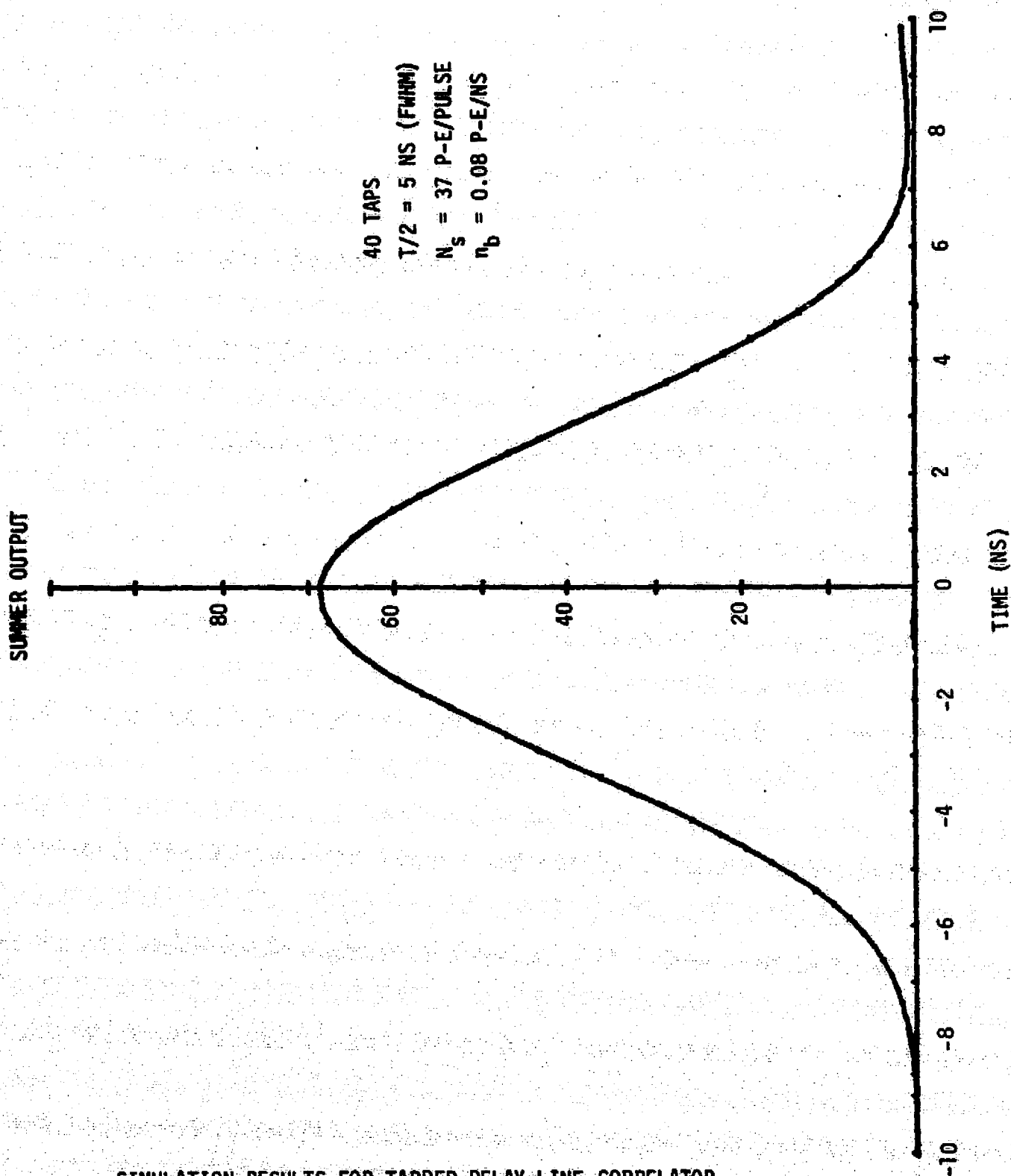


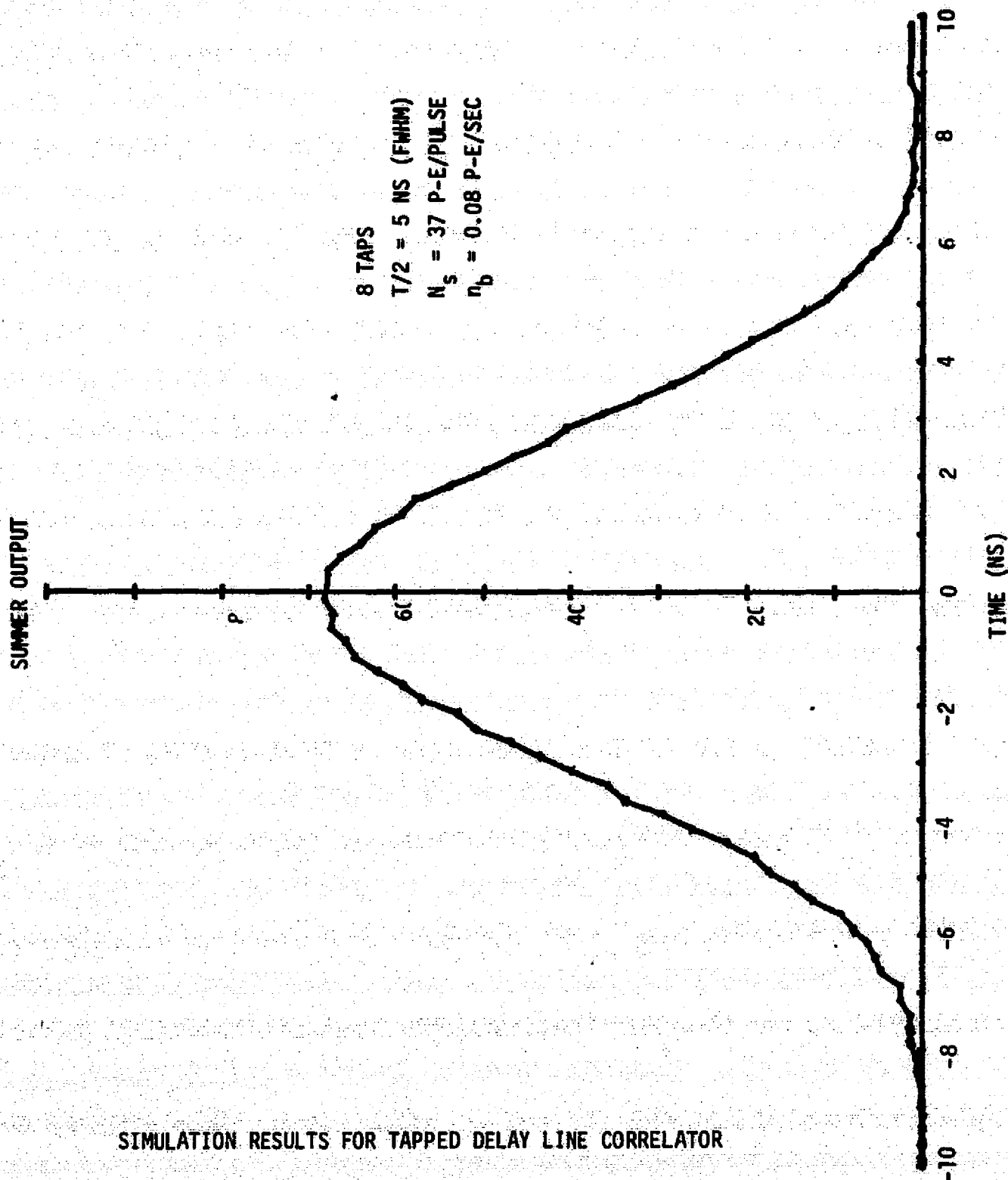
Figure 17



SIMULATION RESULTS FOR TAPPED DELAY LINE CORRELATOR

Figure 18

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SIMULATION RESULTS FOR TAPPED DELAY LINE CORRELATOR

Figure 19

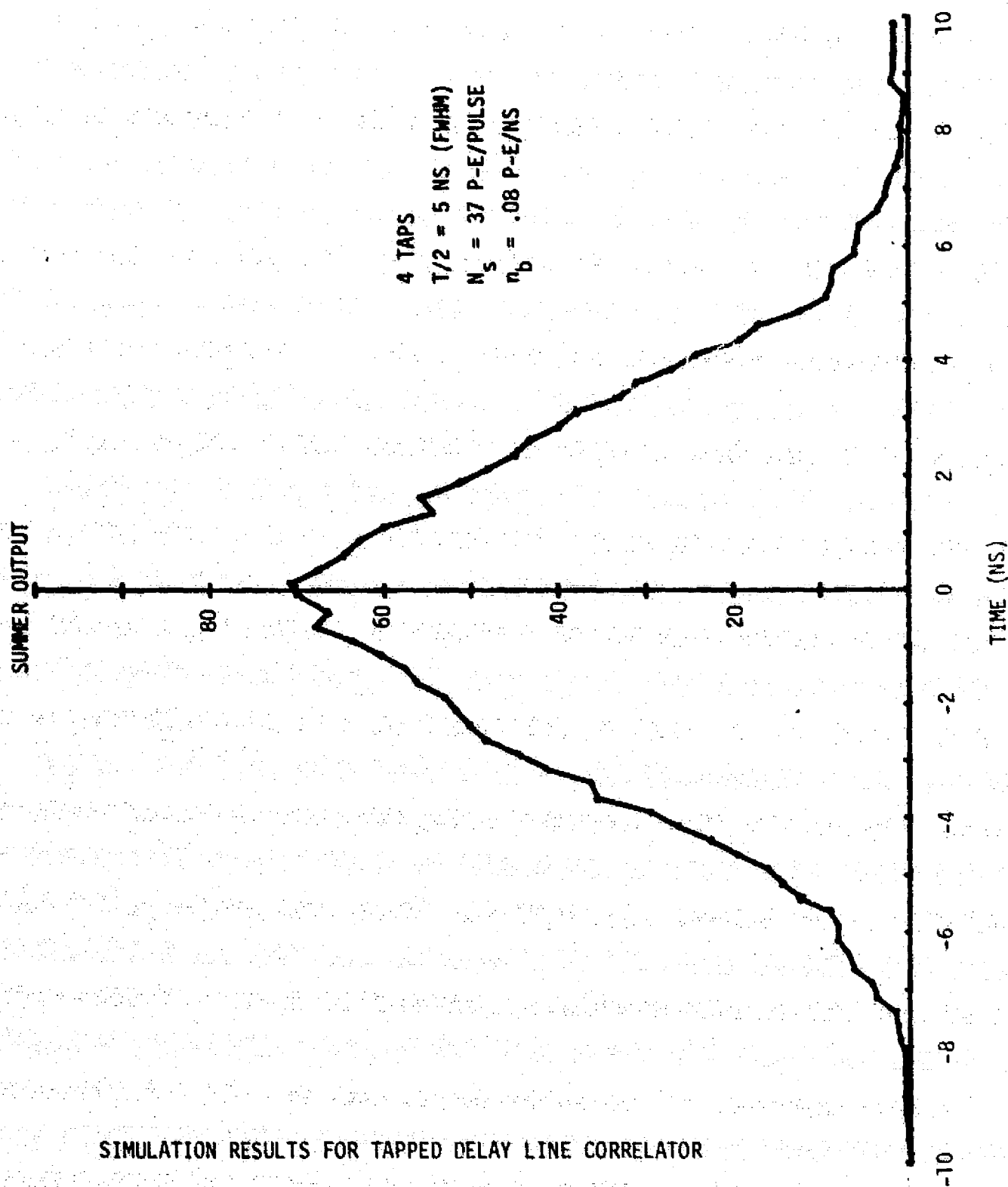


Figure 20

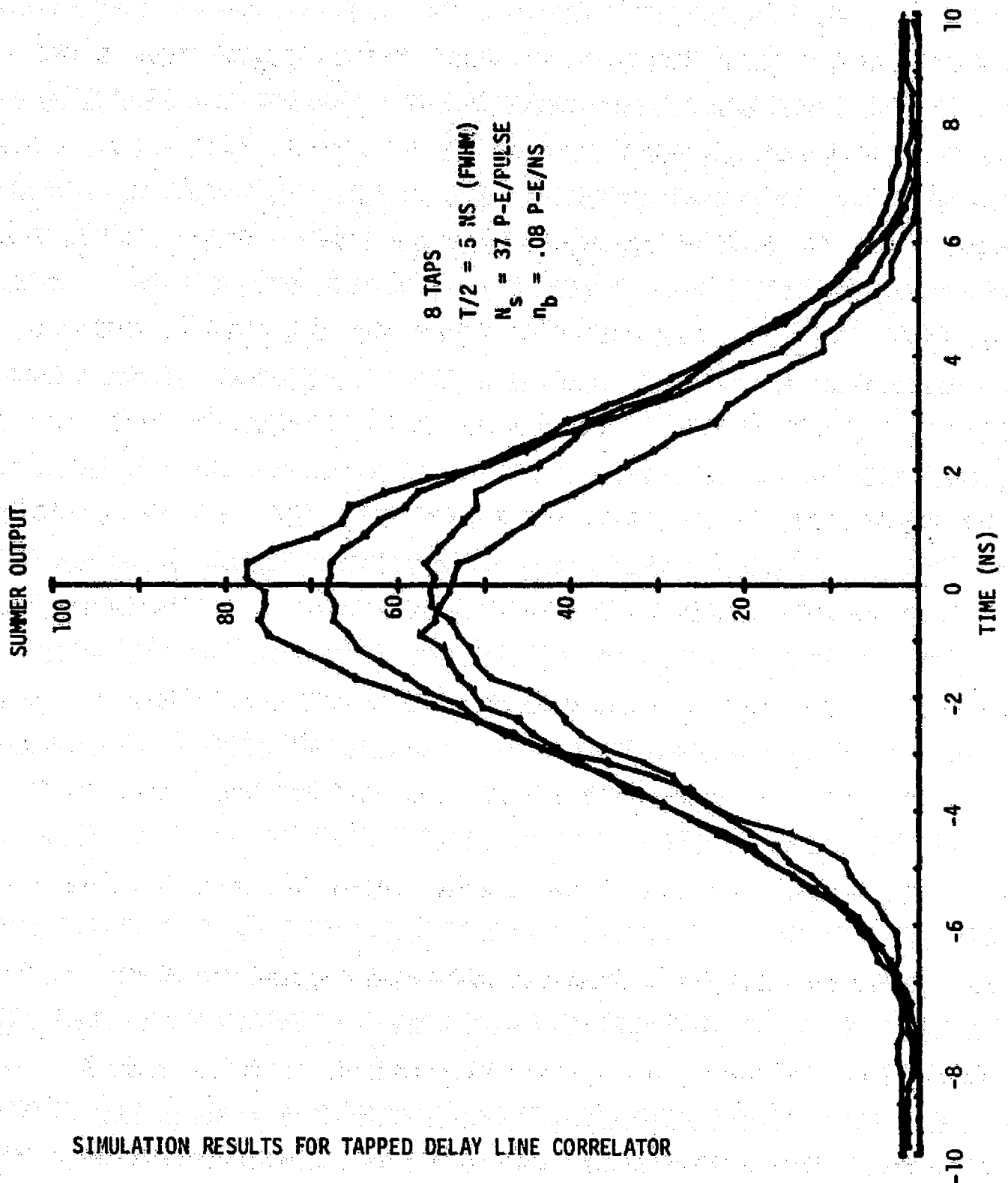
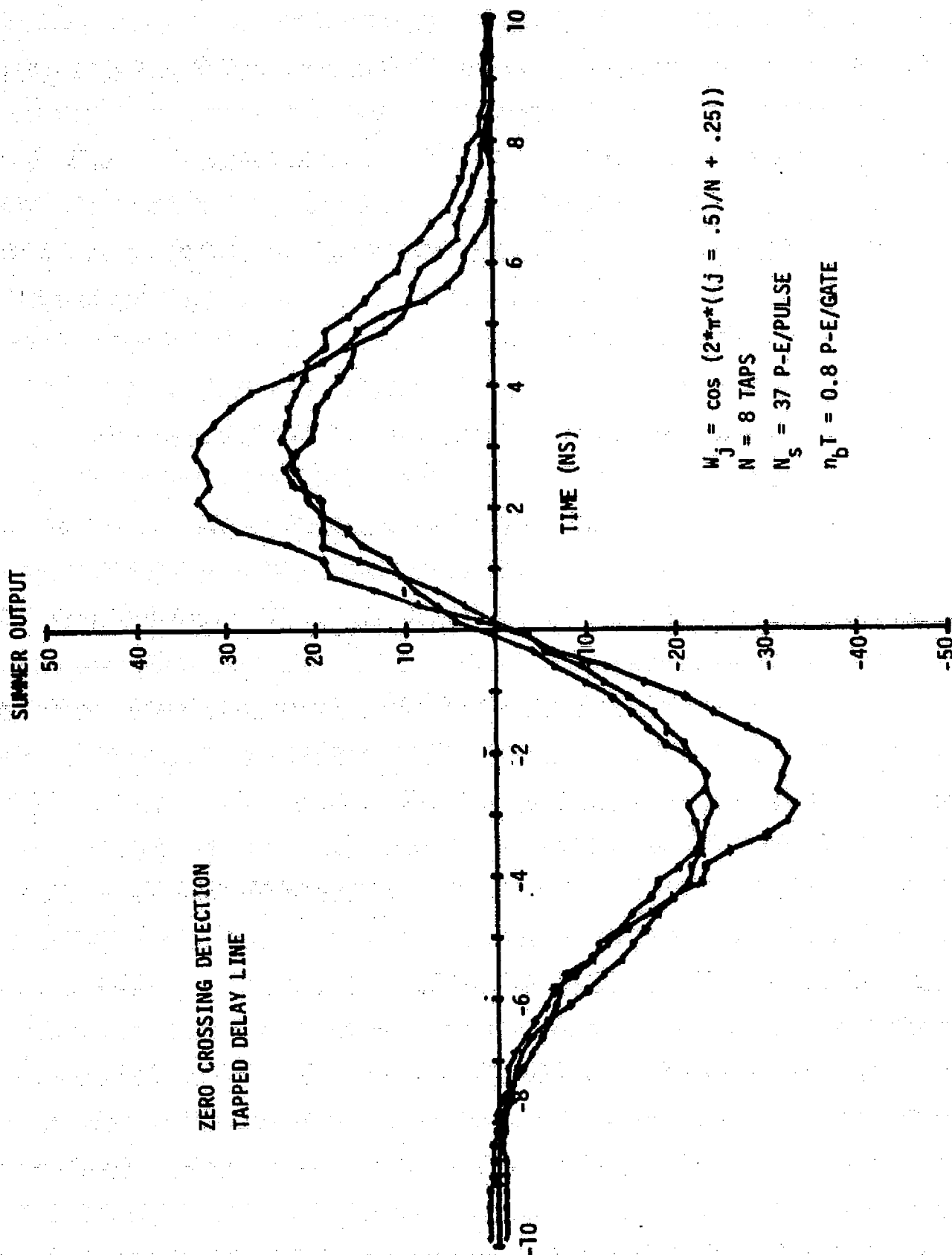


Figure 21



ZERO CROSSING DETECTION
TAPPED DELAY LINE

SIMULATION RESULTS FOR ALTERNATE TAPPED DELAY LINE

Figure 22

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the threshold detector is a bit more laborious in this case, since the pulse is not completely within the delay line, and the high order Bessel functions do not reduce to zero.

Since we previously found that 8 or more taps was essentially equivalent to an ideal matched filter, we will analyze the "continuous" case. The filter output where only background is present is described by the second moment generating function.

$$\psi_B(s) = \int_0^T n_b [e^{s \cos(2\pi\alpha/T)} - 1] d\alpha \quad (35)$$

$$\psi_B(s) = n_b T [I_0(s) - 1]$$

Hence,

$$n_B = 0$$

$$\sigma_B^2 = n_b T/2$$

When only signal is present, we find,

$$\psi_S(s, t) = \int_{t-T/2}^{T/2} n_s [1 + \cos(2\pi(t - \alpha)/T)] [e^{s \cos(2\pi\alpha/T + \pi/2)} - 1] d\alpha \quad (36)$$

Expanding the exponential in a Bessel function series, and performing the trigonometric integrations results in,

$$\begin{aligned} \psi_S(s, t) = n_s T \bigg\{ & [I_0(s) - 1] (1 - t/T + \frac{1}{2\pi} \sin 2\pi t/T) \\ & - I_1(s) [(1 - t/T) \sin 2\pi t/T + \frac{1}{\pi} (1 - \cos 2\pi t/T)] \\ & + \frac{1}{\pi} \sum_{K=2}^{\infty} \frac{(-1)^K I_K(s)}{K(K^2 - 1)} \left[[K^2 (1 - \cos 2\pi t/T) \right. \\ & - (1 - \cos 2\pi Kt/T)] \sin K\pi/2 \\ & \left. + [\sin 2\pi Kt/T - K \sin 2\pi t/T] \cos K\pi/2 \right] \bigg\} \quad (37) \end{aligned}$$

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The maximum expected negative swing occurs near $t = T/4$ (0.284852 T is closer).

$$\begin{aligned} \psi_s(s, T/4) = & \left(\frac{3}{4} + \frac{1}{2\pi}\right) n_s T [\bar{I}_0(s) - 1] - \left(\frac{3}{4} + \frac{1}{\pi}\right) n_s T \bar{I}_1(s) + \frac{n_s T \bar{I}_2(s)}{3\pi} \\ & + \frac{n_s T \bar{I}_3(s)}{3\pi} - \frac{n_s T \bar{I}_4(s)}{15\pi} - \frac{n_s T \bar{I}_5(s)}{5\pi} + \end{aligned} \quad (38)$$

Thus,

$$n_s = \frac{1}{2} \left(\frac{3}{4} + \frac{1}{\pi}\right) n_s T \quad (t = T/4)$$

$$\sigma_s^2 = \left(\frac{3}{8} + \frac{1}{3\pi}\right) n_s T$$

$$n_s \approx \frac{3}{2} n_s T$$

$$\sigma_s^2 \approx \frac{5}{2} n_s T$$

Then, the signal to noise ratios for the two filter configurations (for threshold detection) are,

$$\text{SNR (MF)} = (3/2 n_s T)^2 / (5/2 n_s T) = 0.9 n_s T$$

$$\text{SNR (ZC)} = \left[\left(\frac{3}{8} + \frac{1}{2\pi}\right)^2 / \left(\frac{3}{8} + \frac{1}{3\pi}\right)\right] n_s T \approx 0.5931 n_s T$$

Thus, we conclude that threshold detection with the zero-crossing detector configuration requires ~ 1.8 dB more signal than for the matched filter configuration for comparable performance. The zero-crossing detector is easier to implement reliably than a peak detector; the 1.8 dB penalty can be accepted since we expect to normally operate in the large signal region where detection probabilities are not a problem.

Split Gate Sliding Window Integrator

An alternate concept, to evade precision wide bandwidth tapped delay lines, is to use a split gate sliding window integrator, as shown in Figure 23. This device is very similar to the tapped delay line with zero-crossing detector for time of arrival estimation.

SPLIT GATE SLIDING WINDOW INTEGRATOR BLOCK DIAGRAM

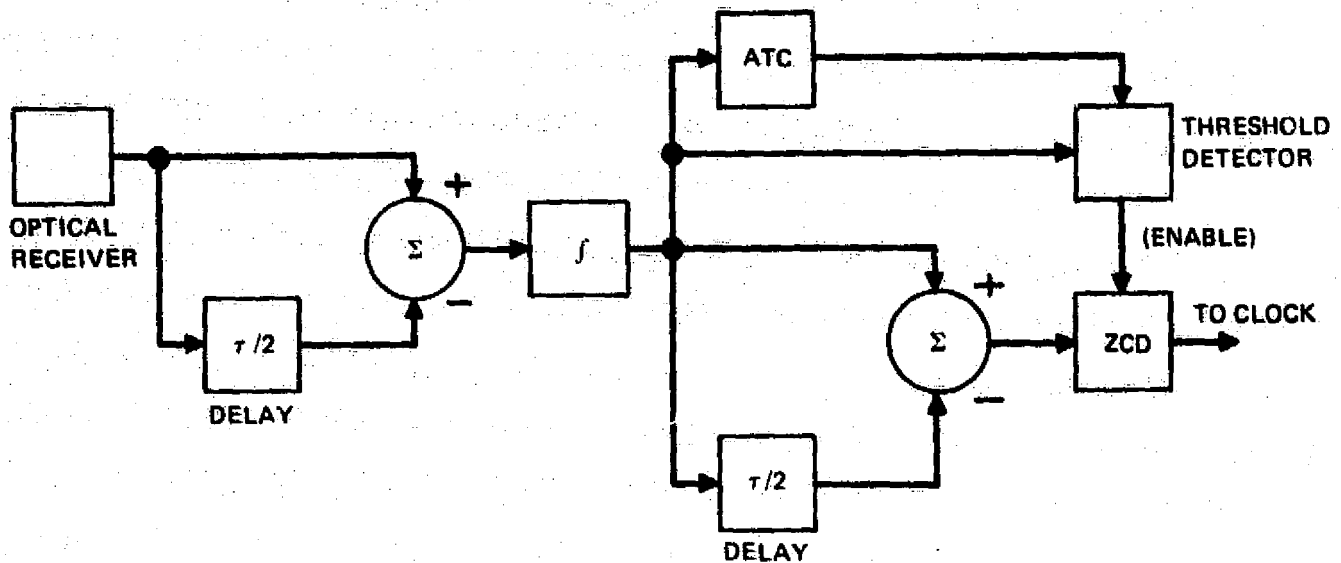


Figure 23

The false alarm and missed detection probabilities are given by the equations derived for the sliding window integrator previously. The performance for time of arrival estimation can be shown to be

$$\overline{E_x^2} = \frac{T^2}{16(\text{SNR})} \quad (39)$$

where $\text{SNR} = \frac{N_s^2}{N_s + n_b T}$

T = total gate width ($2 \times \text{FWHM}$)

n_b = background rate (p-e/sec)

N_s = signal p-e/pulse

and a raised cosine pulse shape was assumed.

For comparison purposes, we note that if a correlator were used,

$$\overline{E_\alpha^2} = \frac{T^2}{2\pi^2(\text{SNR})} \quad (40)$$

If a maximum likelihood estimator were used,

$$\overline{E_\alpha^2} = \frac{T^2}{4\pi^2[N_s + n_b T - \frac{2N_s n_b T + (n_b T)^2}{2}]} \quad (41)$$

At zero background rate, these become,

$$\overline{E_\alpha^2} = \frac{T^2}{16N_s} \Big|_{SG}, \frac{T^2}{2\pi^2 N_s} \Big|_{COR}, \frac{T^2}{4\pi^2 N_s} \Big|_{ML}$$

Under background limited conditions ($n_b T \gg N_s$),

$$\overline{E_\alpha^2} = \frac{T^2(n_b T)}{16 N_s^2} \Big|_{SG}, \frac{T^2(n_b T)}{2\pi^2 N_s^2} \Big|_{COR}, \frac{T^2(n_b T)}{2\pi^2 N_s^2} \Big|_{ML}$$

Thus, the rms TOA error for the split gate tracker is ~ 1.11 times the rms error for a correlator operating on the same signal. The ML estimator is either better than or equal to the correlator, but not by much unless the background rate is very small. Alternatively, we can express the performance in terms of required signal or SNR. In this context, the split gate tracker requires ~ 0.9 dB more signal to shot noise ratio than the correlator for equivalent performance.

2.1.4 Ground Return Pulse

The emitted laser pulse can be reflected from clouds, the intended target, and the ground. In order to determine the relative magnitude of these reflected signals, first consider the signal reflected from the ground. In many cases, the ground is well represented as a diffuse (Lambert) scatterer. The magnitude of the return pulse is then given by,

$$N_G = P_T S_T S_R A_e (r \cos \phi) / \pi R^2 h\nu \quad (42)$$

where

- N_G = scattered signal p-e/pulse
- P_T = transmit energy per pulse (joules)
- S_T = transmittance of transmit path
- S_R = transmittance of receive path
- A_e = receiver effective area (m^2)
- r = reflectance of the ground
- ϕ = zenith angle
- R = range (m)

h = Plank's constant
 ν = optical frequency

If the surface is a more nearly specular scatter at near normal incidence ($\phi=0$),

$$N_S = N_G \times (\pi/4 \theta_e^2)$$

where

θ_e = full planar beamwidth of the scatterer

For example, even moderately choppy water has a significant gain compared to a diffuse reflector at near normal incidence. On the other hand, a concrete road is a nearly Lambertian reflector.

The reflectance coefficient, r , varies considerably. A typical lower limit for black earth is ~ 0.02 . An upper limit is ~ 0.8 , for newly fallen snow.

At normal incidence (looking straight down), the energy in the reflected pulse from new snow is ~ 148 p-e/pulse, which is easily detectable, although considerably smaller than the expected signal from the target at this condition. Thus, a potential false acquisition problem exists if we must reacquire near zenith, or if the target is snow covered. Near the horizon, the return is much smaller (~ 1.6 p-e/pulse) and grossly spread in time, and is, therefore, relatively unimportant.

In order to estimate where the false acquisition possibility ceases to be a problem, consider a laser pulse temporal and spatial envelope given by,

$$S_i(t,r) = a e^{-\left(\frac{t^2}{\tau^2} + \frac{r^2}{r_0^2}\right)} \quad (43)$$

The reflected pulse is then,

$$S_r(t) = \int_0^\infty \int_0^{2\pi} a \exp \left\{ -\frac{r^2}{r_0^2} - \frac{\left(t + \frac{2r}{c} \tan \phi \cos \theta\right)^2}{\tau^2} \right\} d\theta r dr \quad (44)$$

Then, we see,

$$S_r(0) = \int_0^{\infty} \int_0^{2\pi} a \exp -r^2 \left(\frac{1}{r_0^2} + \frac{4 \tan^2 \phi \cos^2 \theta}{C^2 \tau^2} \right) d\theta r dr \quad (45)$$

Interchanging the order of integration yields,

$$\begin{aligned} S_r(0) &= \int_0^{2\pi} \left(\frac{a}{2} \right) \frac{d\theta}{\left(\frac{1}{r_0^2} + \frac{4 \tan^2 \phi \cos^2 \theta}{C^2 \tau^2} \right)} \\ &= \frac{a\pi}{\sqrt{\frac{1}{r_0^2} + \frac{4 \tan^2 \phi}{r_0^2 C^2 \tau^2}}} \\ S_r(0) &= \frac{a\pi r_0}{\sqrt{1 + \frac{4 r_0^2 \tan^2 \phi}{C^2 \tau^2}}} \quad (46) \end{aligned}$$

The total reflected energy is determined to be,

$$N_G(\phi) = \int_{-\infty}^{\infty} S_r(t) dt = a\pi r_0 \pi^{3/2} \quad (47)$$

Thus,

$$S_r(0) = N_G(\phi) / \tau \sqrt{\pi (1 + (2r_0 \tan \phi / C\tau)^2)} \quad (48)$$

Now, $S_r(0)$ is the peak power in the reflected pulse. At normal incidence, all of the reflected pulse would be detectable in a gate width. However, as the zenith angle increases, the pulse is stretched in time, and the maximum energy in the gate is approximately $S_r(0)T$. Thus, we can compute $N_G(\phi)$ from Equation (48), and then compute $S_r(0)T$ to estimate the peak detected signal in the sliding window integrator.

This will yield a good (but conservative) approximation if T , the gate width, is not too much greater than 2τ , and the zenith angle is large enough to have caused a significant decrease in peak intensity. For a 4ns FWHM pulse,

$\tau = 2.4$ ns. Then, for $T = 8$ ns, $h_s = 333$ km, $FOV = .5$ mrad, $r = 0.8$, the estimates of $S_r(0)T$ shown in Table 1.

TABLE 1
SIGNAL BACKSCATTER VS ZENITH ANGLE

ZENITH ANGLE (DEGREES)	$N_C(\phi)$ (P-E/PULSE)	$S_r(0)T$ (P-E/GATE)
0	148.0	148.0
1	147.9	66.9
2	147.7	34.1
5	145.9	13.5
10	139.7	6.35
20	117.05	2.47
30	85.67	1.05

Thus, we see that the energy in a gate width falls off quite rapidly, and becomes virtually insignificant as the zenith angle increases above 10° . At near normal incidence, we could also experience a rise in detected background due to specular scattering. However, the likelihood of conducting ranging measurements at very low zenith angles is quite small at best and can be completely evaded by operational constraints (i.e., look at other targets).

It does appear possible to use the system in an altimeter mode if such is desired, although it would probably be necessary to alter the time of arrival decision strategy, and the transmit beamwidth and receiver field of view for this mode.

Also, since most terrains are about an order of magnitude less reflective than fresh snow, it would probably be necessary to increase the receiver telescope diameter to obtain fairly confident ground return detection. Finally, we note that the mean spacing between illuminated spots for a 10 pps laser is about 740m, compared to a beam diameter of ~ 165 m, thus successive pulses are terrain independent and the range gate would have to be wide enough to cover surface roughness.

In order to test this hypothesis, we estimate the advantage of signal power to solar background power. This is readily accomplished by noting that the solar spectral irradiance in the vicinity of $.53 \mu\text{m}$ is about $0.2 \text{ w/m}^2\text{-}\text{\AA}$. If we use a $5\text{\AA}$ optical filter, and assume 0.5 mrad field of view from a 333 km (180 nmi)

spacecraft, the incident solar power on the ground that is within our field of view and optical bandwidth is ~ 21800 watts. For a 4 ns FWHM, 10 mJ laser pulse, the peak power is 2.5×10^6 watts, thus the peak signal power to mean background power ratio is ~ 115 to 1, and the signal should be detectable above the background in a modestly sized gate width.

Thus, we conclude that ground reflected signal need not be a serious concern if we limit the observation region to elevation angles less than 80 to 85° for normal terrain in the vicinity of the target. Further, it may be possible to configure the experiment to allow operation as an altimeter, when not ranging, at least over favorable terrain.

2.1.5 Laser Ranging Experiment Parameter Selection

Two types of lasers were considered for the laser ranging experiment, referred to as long pulse and short pulse lasers. The long pulse laser is a Q-switched, cavity dumped Nd:YAG laser with an output pulsewidth on the order of 4 to 6 ns (FWHM). The short pulse laser is a Q-switched, mode-locked, cavity dumped laser with an output pulsewidth on the order of 0.1 ns (FWHM). The experiment design approach for these two lasers will differ considerably. The longer pulse laser system design will be driven by the need to obtain 2 to 10 cm rms range measurement accuracy. The short pulse laser system design is almost totally driven by detection statistics; if the return pulse is detected, the theoretical error will be less than 2 cm rms. The two concepts are therefore separately discussed.

Long Pulse Laser Ranging Experiment

In the preceding sections the performance of the laser ranging experiment, expressed in terms of theoretical ranging measurement accuracy, was found to be a function of the signal to shot noise ratio.

$$SNR = \frac{N_S^2}{N_S + N_B} \quad (49)$$

where,

$$N_S = \sigma (E_p G_T) A_e (\eta S_T S_R S_a / (4\pi)^2 R^4 h\nu) \quad (50)$$

Equations (4) and (9) from Section 2.1.1 have been rearranged in (49) and (50) to group transmitters and receiver variables, and the more or less fixed parameters.

The transmitter variables are the energy per pulse (E_p) and the transmit antenna gain (G_T), which is a function of the transmit beam width ($32/\alpha_T^2$). The receiver variables are the receiver aperture effective area (A), the optical filter bandwidth ($\Delta\lambda$), and the receiver field of view ($\Omega_R = \frac{\pi}{4} (\text{FOV})^2$). The target radar cross-section (σ) is also a variable for this study. The remaining parameters are largely determined by the state-of-the-art and the environment.

The transmit beamwidth and the receiver field of view are primarily constrained by the achievable pointing accuracy. Let ρ be the maximum design transmitter pointing error and let α_T be the transmit beamwidth at the e^{-2} power points. The weakest design signal detected would be proportional to $\alpha_T^{-2} e^{-8\rho^2/\alpha_T^2}$. This would be maximized if $\alpha_T = \rho\sqrt{8}$.

The transmitter pointing error consists of a random component, which can be considered as a pulse to pulse variable, and a "static" component, which is relatively constant during a pass. If we assume the random component consists of two orthogonal, normal, random variables with equal variances, σ_p , there the probability of the pointing error exceeding ρ is given by,

$$P_r \{>\rho\} = Q(\eta/\sigma_p, \rho/\sigma_p) \quad (51)$$

where η = static pointing error

and $Q(\alpha, \beta)$ = Markum Q function.

$$= \sum_{k=0}^{\infty} \left(\frac{\alpha}{\beta}\right)^k I_k(\alpha, \beta) e^{-(\alpha^2 + \beta^2)/2}$$

The Markum Q function is relatively easy to evaluate numerically, but we seek an insight into the relationship between η , ρ , and σ_p . For a fixed value of $Q(\alpha, \beta) = 0.1$ (one of every 10 pulses is weaker than predicted), a reasonable approximation is,

$$\rho = \beta_o \sigma_p + \sqrt{\sigma_p^2 + \eta^2} \quad (52)$$

where $\beta_o = 1.12$

For $Q(\alpha, \beta) = 0.01$, the approximation is the same form but $\beta_o = 2.0$. For $Q(\alpha, \beta) = 0.001$, use $\beta_o = 2.72$. Assume $\sigma_p = 0.05$ mrad, $\eta = 0.09$ mrad. Then, a value of $\rho = 0.159$ mrad will result in 9 of 10 pulses at nominal or greater signal strength, and $\alpha_T = 0.45$ mrad would be optimal. For 99 of 100 pulses at nominal, $\rho = 0.203$ mrad, and $\alpha_T = 0.574$ mrad would be optimal. A value of $\alpha_T = 0.5$ mrad is a reasonable compromise.

The receiver field of view needs to be somewhat greater than twice the selected value of ρ , since there is some net pointing error between the receiver and the transmitter. For preliminary design purposes it is, therefore, reasonable to choose the receiver field of view equal to the transmit beamwidth.

The "fixed" link parameters include quantum efficiency, the transmit and receive optical efficiencies, atmospheric transmissability, the range, and the background radiance. At $0.532\mu\text{m}$, a 25% quantum efficiency is achievable in a high speed photomultiplier. The values of $S_T = 0.8$ and $S_R = 0.4$ have been assigned to allow some margin for degradation. The atmospheric transmissability is a function of the optical thickness (T^1) of the atmosphere and the elevation angle. For targets at or near sea level, an optical thickness of 0.425 at $0.532\mu\text{m}$ corresponds to clear day conditions. Then, $S_a = \exp\{-2T^1/\sin E\}$, where E is the elevation angle. The Shuttle altitude is assumed to be 333 km (180 nmi), and the background radiance, $0.017 \text{ w/m}^2\text{-A-ster.}$, corresponds to sun-lit earth viewed from space. The design minimum elevation angle was chosen to be 20° , resulting in $S_a = 0.0833$ and $R = 838$ km.

The remaining variables, E_p , P , A_e , and $\Delta\lambda$ can be chosen to yield any desired theoretical ranging accuracy requirement. Assume a 10 cm rms accuracy requirement, using the split gate sliding window integrator technique. The required $\text{SNR} = 14$ (11.46dB). Figure 24 shows the required receiver aperture diameter to achieve this SNR as a function of the product of energy per pulse and target radar cross-section for various optical filter bandwidths. Note that the receiver diameter is inversely proportional to the required rms ranging accu-

racy, thus to achieve 2 cm rms the receiver diameter would have to be 5 times greater than shown in Figure 24. An upper limit of 19 cm diameter (7.5 inches) was considered reasonable for the receiver. As mentioned previously, a 50 mj/pulse laser is considered achievable, and a target radar cross-section of 10^7 m^2 is considered feasible. These limits are also shown in Figure 24, to emphasize the available design margin.

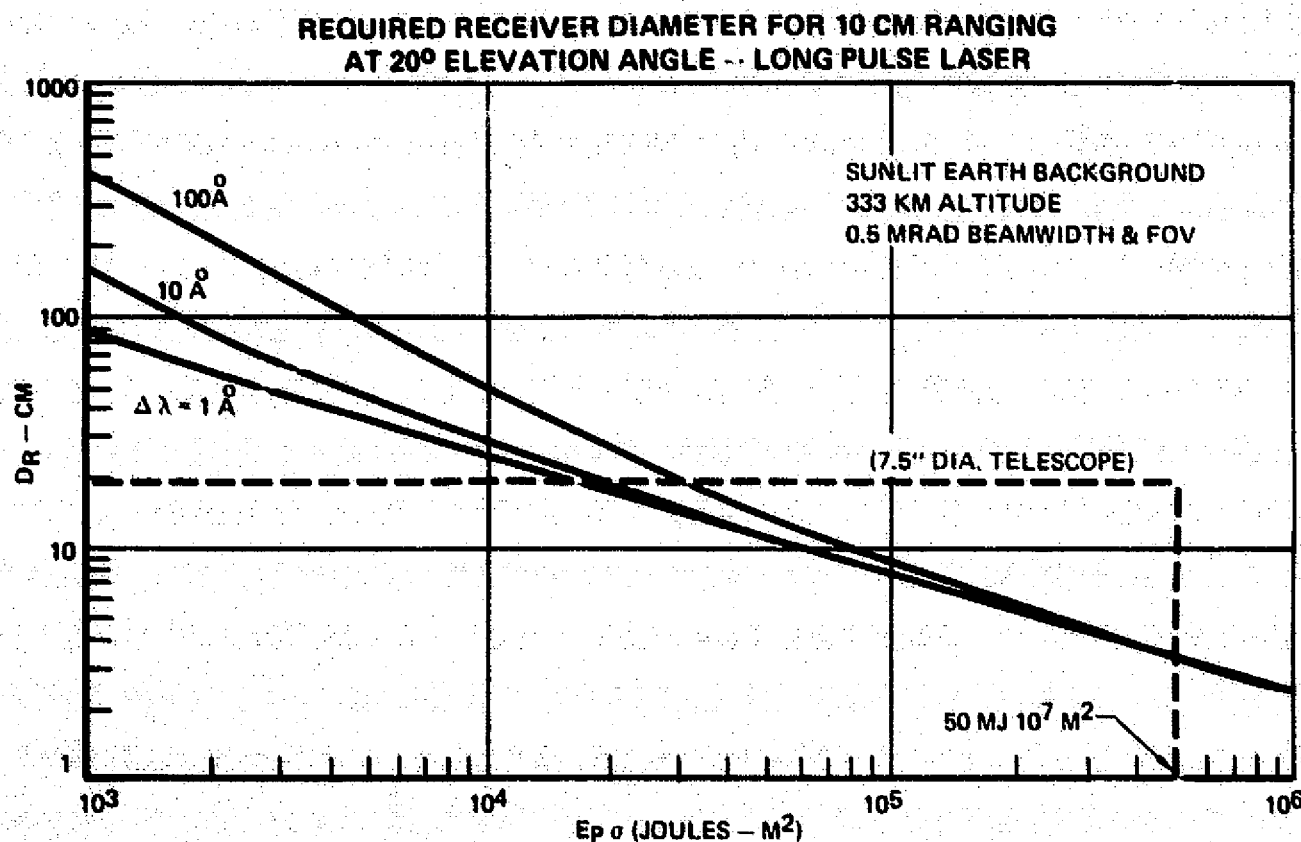


Figure 24

In order to meet the minimum acceptable performance, 10 cm rms with a 19 cm diameter receiver aperture, the $E_p \sigma$ product must be at least 1.5×10^4 joules- m^2 , assuming a one Angstrom optical filter bandwidth. If $\sigma = 10^7 \text{ m}^2$, a 1.5 mj/pulse laser would suffice (zero margin). Alternatively, for a 50 mj/pulse laser and $\sigma = 10^7 \text{ m}^2$, a 3.5 cm (1.4 inch) diameter receiver aperture would yield 10 cm rms (zero margin) ranging accuracy. Note that the required $E_p \sigma$ is affected by the minimum elevation angle for ranging. For example, increasing the minimum elevation angle to 30° would reduce the required $E_p \sigma$ product by a factor of

7.2 (3.6 dB). Decreasing the minimum elevation angle to 10° would increase the required $E_p \sigma$ product by a factor of 56.1 (17.5 dB).

The Shuttle altitude also affects the required $E_p \sigma$ product. At 20° minimum elevation angle, increasing the Shuttle altitude to 500 Km would increase the required $E_p \sigma$ product by a factor of 4.1 (6.1 dB).

The selected design values, 19 cm diameter receiver aperture and 5×10^5 joules-m², provide greater than 10 dB margin over the minimum requirements, which is considered a comfortable margin for preliminary design.

Table 2 summarizes the link margin for the selected system parameters.

Short pulse laser ranging experiment - The major concern for the short pulse laser ranging experiment is the detection statistics; if the pulse is detected, the theoretical time of arrival estimation error is well within the most ambitious goal. We selected a 0.5×10^{-3} false alarm probability goal, and a 99% detection probability goal, and parametrically evaluated the effects of optical filter bandwidth and the energy/pulse-target cross-section product on the required receiver aperture diameter, as shown in Figure 25. The results are quite similar to the results for the long pulse laser ranging experiment discussed previously, except that the optical filter bandwidth effects are apparent even with relatively large energy-cross section products. Table summarizes the link margin analysis for a specific point design similar to the design for the long pulse laser ranging experiment.

2.2 Mission Simulation

The question of acquisition and tracking cannot be treated in isolation. Experiment characteristics, especially the statistics of the errors in results, is affected by selection of the pointing system. In the design phase, the experiment behavior can be used to assist selection of fundamental pointing system parameters. Among these are sample rates and allowable acquisition times. A computer simulation of the experiment is necessary to the design of the laser pointing system.

The experiment has an estimation procedure associated with it. This in fact, is the very essence of the experiment. Because of the complexity of the system, the large number of corner reflectors which may be considered and the extreme

TABLE 2
LINK MARGIN ANALYSIS
LONG PULSE LASER

<u>PARAMETER</u>	<u>VALUE</u>	<u>COMMENTS</u>
1. Transmit Energy	-13.01 dBJ	50 mj/pulse
2. Transmit Losses	-0.97 dB	80% Transmittance
3. Transmit Antenna Gain	81.07 dB	0.5 mrad beamwidth
4. Pointing Loss	-4.34 dB	
5. Free Space Loss	-265.93 dB	838 KM
6. Atmospheric Loss (Down)	-5.40 dB	0.425 Optical Thickness @ 20° Elev.
7. Target Gain Product	206.47 dBJ	10^7 m^2
8. Atmospheric Loss (Up)	-5.40 dB	
9. Free Space Loss	-265.93 dB	
10. Receive Antenna Gain	121.00 dB	19 cm Dia.
11. Receiver Losses	-3.98 dB	40% Transmittance
12. Received Signal Energy	-156.42 dBJ	
13. Energy/Photon	-184.26 dBJ	0.532 μm
14. Received Photons/Pulse	27.84 dB	
15. Quantum Efficiency	-6.02 dB	25%
16. Received P-E/Pulse	21.82 dB	152. p-e/Pulse
17. Background Radiance	-17.70 dBW	$0.017 \text{ W/M}^2\text{-}\overset{\circ}{\text{A}}$ Ster.
18. Receiver FOV (Sterrad.)	-67.07 dB	0.5 mrad
19. Optical Filter Bandwidth	6.99 dB	5 $\overset{\circ}{\text{A}}$
20. Receive Antenna Area	-15.47 dBm^2	
21. Receiver Losses	-3.98 dB	
22. Received Background Power	-97.23 dBW	
23. Energy/Photon	-184.26 dBJ	
24. Received Photons/Sec.	87.03 dB	
25. Quantum Efficiency	-6.02 dB	
26. Received P-E/Sec	81.01 dB	$1.26 \times 10^8 \text{ p-e/sec}$
27. Receiver Gate Width	-80.00 dB	10 Nanoseconds
28. Background P-E/Gate	1.01 dB	1.26 p-e/Gate
29. Received P-E/Pulse	21.82 dB	152 p-e/Pulse
30. Required P-E/Pulse	11.83 dB	10 cm rms
31. Link Margin	9.99 dB	

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TABLE 3
LINK MARGIN ANALYSIS
SHORT PULSE LASER

<u>PARAMETER</u>	<u>VALUE</u>	<u>COMMENTS</u>
1. Transmit Energy	-16.99 dBJ	20 mj/pulse
2. Transmit Losses	-0.97 dB	80% Transmittance
3. Transmit Antenna Gain	81.07 dB	0.5 mrad beamwidth
4. Pointing Loss	-4.34 dB	
5. Free Space Loss	-265.93 dB	838 KM
6. Atmospheric Loss (Down)	-5.40 dB	0.425 Optical Thickness @ 20° Elev.
7. Target Gain Product	206.47 dB	10^7 m^2
8. Atmospheric Loss (Up)	-5.40 dB	
9. Free Space Loss	-265.93 dB	
10. Receive Antenna Gain	121.00 dB	19 cm Dia.
11. Receiver Losses	-3.98 dB	40% Transmittance
12. Received Signal Energy	-160.40 dBJ	
13. Energy/Photon	-184.26 dBJ	0.532 μm
14. Received Photons/Pulse	23.86 dB	
15. Quantum Efficiency	-6.02 dB	25%
16. Received P-E/Pulse	17.84 dB	60.8 p-e/Pulse
17. Background Radiance	-17.70 dBW	0.017 $\text{W}/\text{M}^2 \text{ } ^\circ \text{A-Ster.}$
18. Receiver FOV (Sterrad)	-67.07 dB	0.5 mrad
19. Optical Filter Bandwidth	6.99 dB	5 $\text{ } ^\circ \text{A}$
20. Receive Antenna Area	-15.47 dBm^2	
21. Receiver Losses	-3.98 dB	
22. Received Background Power	-97.23 dBW	
23. Energy/Photon	-184.26 dBJ	
24. Received Photons/Sec.	87.03 dB	
25. Quantum Efficiency	-6.02 dB	
26. Received P-E/Sec	81.01 dB	
27. Receiver Gate Width	-96.99 dB	0.2 ns
28. Background P-E/Gate	-15.98 dB	0.025 p-e/Gate
29. Received P-E/Pulse	17.84 dB	60.8 p-e/Pulse
30. Required P-E/Pulse	10.93 dB	12.4 p-e/Pulse
31. Link Margin	6.91 dB	

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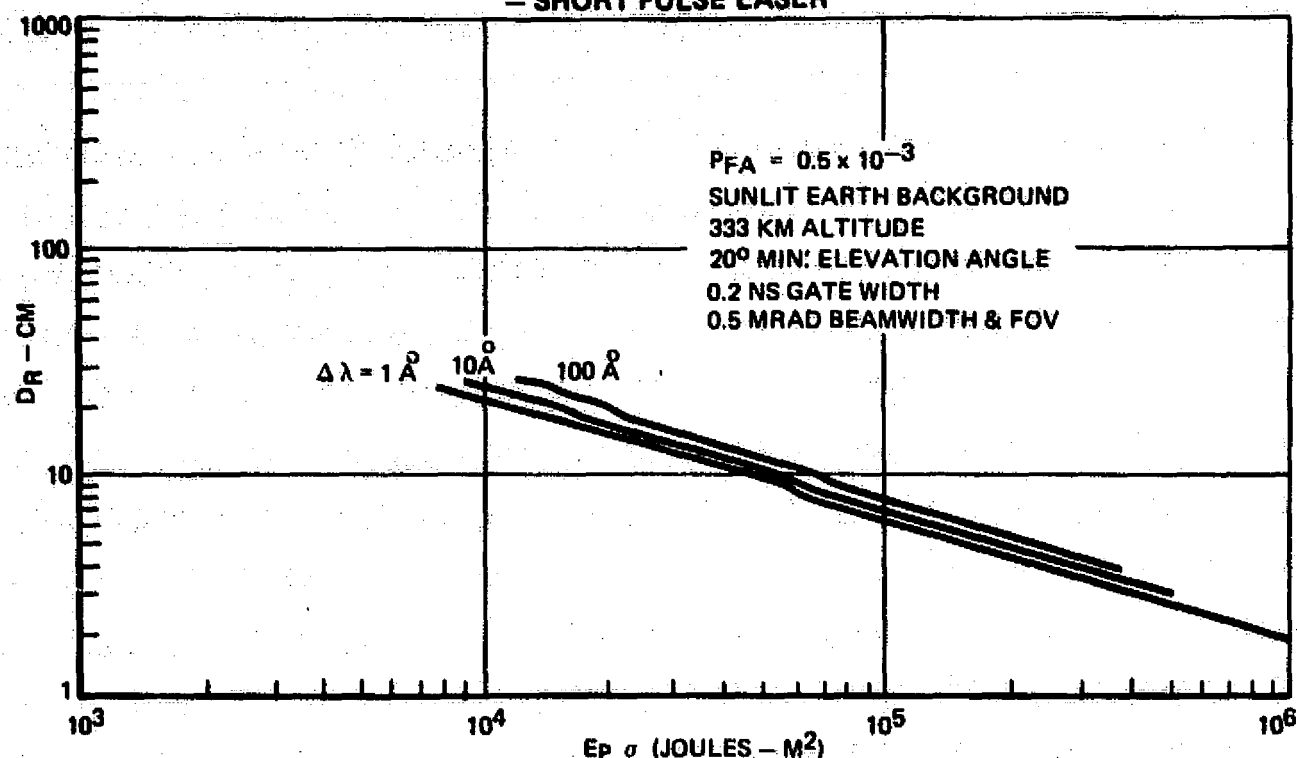
REQUIRED RECEIVER DIAMETER FOR 99% DETECTION PROBABILITY
— SHORT PULSE LASER

Figure 25

accuracies desired, the estimation procedure is a very involved and complicated program. Mathematical simulations of this exist but are prohibitively expensive in computer time and in cost to be used for engineering design. Because of this a simplified simulation was conceived so that almost any effect can be included in the statistics if necessary even though the dynamics of the nominal trajectory cannot be used for prediction purposes. This simulation has been implemented on a 6500 Cyber computer.

2.2.1 Program Description

The basic kinematics of the simulation represents a spacecraft travelling in a circular orbit with specified altitude, node, inclination and anomaly from the node at time zero. This was implemented by assuming perfectly circular motion with uniform velocity. This was to avoid any necessity of numerical integration in the simulation. The array of reflectors on the Earth's surface were simulated by circular motion representing the earth's rotation. The position of any reflector at any specified time is obtained by calling a subroutine. In this

subroutine the positions of every reflector at time zero are calculated just once when the subroutine is first called. The program is so dimensioned that any number of reflectors can be considered. Rearrangement of reflector grid geometries is effected by simple modification of the subroutine.

Simulation of the sampling is also handled by means of a subroutine. This selects the reflector which is to be observed in accordance with any observation policy which one may wish to investigate. The time of the observation is also provided in accordance with any sample rate specifications. Acquisition delays are taken into consideration by having the routine suppress observations for a specified time after the acquisition reflector has arrived within view.

The reflector grid selected for this study consisted of five reflectors situated at the corners of a 25 km square with one reflector at the center which is at latitude 33 deg north with the sides of the square situated east-west, north-south. The purpose of the experiment is to determine the relative position of these reflectors each with respect to another. Four "outrigger" reflectors were added. It is not required to determine the positions of these with high precision. These were added to investigate their ability to aid in spacecraft position determination. These four reflectors were positioned at the corners of a 200 km square with either its sides north-south, east-west or its diagonals north-south, east-west.

With these reflector arrangements the measurement policy adopted was to range upon the most remote outrigger which is in view (in view meaning that the line-of-sight is greater than 20 degrees in elevation) if any of the target grid reflectors are not in view. If the target reflectors are in view, then the range to each of them is measured in succession in batches of five measurements. After this the outriggers are observed once each and the process is then repeated. The intervals between observations are one second unless the observation changes to a new reflector, then it is five seconds. An acquisition time of ten seconds was assumed. Subsequent evaluation indicated that these assumptions are adequate for the laser ranging system design.

A gimbal arrangement was assumed. This assumed that the system was aboard a space shuttle which had its longitudinal axis directed normal to the orbit plane and rolled to an attitude so that at the time of closest approach to the central ground reflector, the roll gimbal was at its neutral position. Computations of

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gimbal angle time histories was provided in a subroutine. Gimbal rates at sample times were also computed as was range, range rate and the magnitudes of the corrections in return pulse arrival time due to atmospheric refraction. If E is the elevation angle and $A = 2.4\text{m}$, $B = 0.0025\text{m}$ then the range correction is given by

$$\Delta R = A/\sin E - B/\sin^3 E$$

and the atmospheric time of arrival by

$$2\Delta R/c$$

and the first order velocity correction by

$$2 \frac{\rho}{c} \frac{\dot{\rho}}{c}$$

where ρ is range and c is the velocity of light.

2.2.2 Statistics

In order to describe the statistics of the estimation procedure which constitutes the experiment we must specify the state vector which includes all of the variables pertinent to the problem. We include the three position components of each reflector and the position and velocity of the spacecraft. If there are M reflectors then we have specified $3M + 6$ variables so far. To these we shall add five geopotential terms. According to our assumption of a purely circular orbit these geopotentials are considered to be nominally zero. Their purpose is only to degrade the statistics of the spacecraft state. The range measurements are considered to have an error which is nearly systematic. An autocorrelation function of the form

$$R(\tau) = \sigma_B^2 e^{-|\tau|/T} \quad (53)$$

was assumed with $T = 20$ seconds. This can represent attitude fluctuation of the spacecraft. The range is also assumed to have a random error which is uncorrelated between samples. The nearly systematic error is appended to the state vector. Also appended is a purely systematic error in the longitudinal component of the centered mass of the spacecraft. Thus, the total number of state components is now $3M + 13$.

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2.2.3 Initial Covariance

The initial covariance for the target location will depend upon the surveying accuracies with respect to the dynamical center of the spacecraft orbit. We assume that the center of the grid will be known only to within a spherical distribution with a standard deviation of $\sigma_0 = 100$ ft. Relative to this bench mark the individual reflectors were assumed to have a circular distribution which is uncorrelated between reflectors with standard deviation $\sigma_T = .3$ ft. Thus, the initial covariance for the reflectors is of the form



The initial covariance of the spacecraft state which was used corresponds to good tracking (i.e., within two orbit periods) by a ground network. This is a complete six by six matrix in orbit plane coordinates. This matrix is as follows (in ft and ft/sec).*

*Reference (1) Lyndon B. Johnson Space Center Memorandum to FM3/Mission Analysis Branch from FMS/Mathematical Physics Branch, dated 29 May 1975.

.275671+05	-.802681+05	.294705+04	.932597+02	-.177456+02	-.126507+01
	.799405+06	-.191430+05	-.795606+03	.575508+02	-.106914+03
		.116397+05	.182926+02	.128867+01	-.334140+01
			.803689+00	-.680565-01	.977056-01
Symmetric				.240196-01	.136106-02
					.525264-01

The covariance of the spacecraft state is reinitialized to this at the beginning of every pass over the target grid area.

The geopotential terms considered were C(0,0), S(7,4), C(6,5), C(7,6) and S(5,5). These were selected by computing the disturbance acceleration acting upon the spacecraft by a 1σ value of the expected error in the known values of each of the harmonics up to C(7,7), S(7,7) acting individually. Those selected gave the largest acceleration while the spacecraft was closest to the reflector grid. The 1σ value of the errors in these were considered the apriori standard deviations. These are

$$\begin{aligned}\sigma &= 0.1 \times 10^{-5} \text{ for } C(0,0), \\ &0.5 \times 10^{-10} \text{ for } S(7,4), \\ &0.1 \times 10^{-9} \text{ for } C(6,5), \\ &0.1 \times 10^{-10} \text{ for } C(7,6), \\ &0.3 \times 10^{-9} \text{ for } S(5,5)\end{aligned}$$

Great accuracy in these quantities is not critical to the simulation since these are used merely to couple with the spacecraft state components. This is needed to prevent a too optimistic spacecraft state component determination.

Initial covariance for the range bias was assumed represented by $\sigma_B = .1$ ft and for the spacecraft center of mass by $\sigma_C = .5$ ft.

2.2.4 Covariance Propagation

The initial covariances which have been discussed must, in order to be used by the estimation procedure, be propagated to the time at which a range measurement is taken and also propagated between measurements. For the reflector positions this involves only the kinematics of the Earth rotation. For the spacecraft state this must involve circular orbit state transition matrices and the coupling from the geopotential effects. The geopotential effects are considered stationary. For the range bias the range error autocorrelation function must

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be taken into account. The spacecraft center of mass error is stationary. It is desired that numerical integration of Riccati equations is to be avoided.

Earth rotation influence upon the reflector covariance is given simply by

$$\begin{pmatrix} \cos \omega_E \Delta t & -\sin \omega_E \Delta t & 0 \\ \sin \omega_E \Delta t & \cos \omega_E \Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix} S_R \begin{pmatrix} \cos \omega_E \Delta t & -\sin \omega_E \Delta t & 0 \\ \sin \omega_E \Delta t & \cos \omega_E \Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix}^T$$

where ω_E is the Earth rotation rate, Δt is the time interval over which the reflector covariance S_R must be propagated.

The spacecraft state covariance in the absence of geopotential effects is propagated by means of the circular orbit state transition matrix ϕ_o given by

$$\phi_o = \begin{bmatrix} 2-c & s & 0 & \frac{s}{\omega} & \frac{2}{\omega}(1-c) & 0 \\ -3\omega\Delta t + 2s & 2c-1 & 0 & \frac{2}{\omega}(1-c) & \frac{1}{\omega}(-3\omega\Delta t+4s) & 0 \\ 0 & 0 & c & 0 & 0 & \frac{s}{\omega} \\ -\omega(-3\omega\Delta t+s) & \omega(1-c) & 0 & 2-c & -(-3\omega\Delta t+2s) & 0 \\ -\omega(1-c) & -\omega s & 0 & -s & 2c-1 & 0 \\ 0 & 0 & -\omega s & 0 & 0 & c \end{bmatrix} \quad (54)$$

Here ω is orbital angular velocity, $c = \cos \omega\Delta t$, $s = \sin \omega\Delta t$, and the coordinates are respectively attitude, downrange distance, normal to orbit plane distance and their time derivatives. The propagated spacecraft state covariance is

$$\phi_o S_o \phi_o^T$$

where S_o is the initial spacecraft state covariance.

To include the coupling of the geopotential uncertainties a matrix ψ_o was considered. This is given by

$$\psi_o = \int_0^{\Delta t} \phi(\zeta) d\zeta \begin{pmatrix} 3 \times 5 \\ 0 \\ 3 \times 5 \\ \frac{\partial a}{\partial h} \end{pmatrix} \quad (55)$$

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here a is the spacecraft acceleration and h represents the geopotential terms. This is derived in Appendix A. In this case the propagation of both the spacecraft and geopotential terms is given by

$$\begin{pmatrix} \phi_o & \psi_o \\ 0 & I \end{pmatrix} S \begin{pmatrix} \phi_o & \psi_o \\ 0 & I \end{pmatrix}^T$$

Covariance propagation for the range error involves the autocorrelation function

$$R(\tau) = \sigma_B^2 e^{-|\tau|/T}$$

In this case a propagation over time Δt requires that the covariance of the range error be modified by first multiplication by $e^{-2 \Delta t/T}$ and then increased by addition of the term $\sigma_B^2 (1 - e^{-2 \Delta t/T})$. Furthermore every correlation term between the range error and the state must be attenuated by multiplication by $e^{-\Delta t/T}$. One can see that over a long time interval the range error covariance is nearly reinitialized.

The covariance of the spacecraft center of mass error is as mentioned previously, not modified during state propagation.

Unmodelled drag effects were also included. These were treated in much the same manner as the geopotential terms with the exception that no state variables were added to the system. An unknown but nominally zero coefficient of an acceleration acting opposite to the spacecraft velocity was considered. An additional spacecraft state covariance was computed based upon the covariance of this drag coefficient. This term is given by

$$\Delta S = [\psi(t_2) - \phi(t_2)\phi^{-1}(t_1)\psi(t_1)]\sigma_k^2[\psi(t_2) - \phi(t_2)\phi^{-1}(t_1)\psi(t_1)]^T \quad (56)$$

where $\psi(t)$ is now the coupling term expressing the influence of the drag term acting from the initialization time of the overfly pass to the time t . The term ΔS is the additional covariance due to unmodelled drag which has been accrued in the time interval beginning at t_1 and ending at t_2 . These computations have been relegated to a subroutine.

The estimation procedure which incorporates the measurements and modify the

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state error covariance was implemented as a Kalman filter. The calculations pertinent to the covariance simulation involve the state covariance S^- prior to the measurement update, the variance of the random error on the (range) measurement σ_M^2 and the "observability" matrix M .

$$M = \left(\frac{\partial \text{state}}{\partial \text{range}} \right)^T \quad (57)$$

With this nomenclature the updated covariance S^+ is given by the Kalman formula

$$S^+ = S^- - S^- M^T (M S^- M^T + \sigma_M^2)^{-1} M S^- \quad (58)$$

Some manipulations in matrix algebra shows that this formula is mathematically equivalent to the least squares formula

$$(S^+)^{-1} = (S^-)^{-1} + M^T \frac{1}{\sigma_M^2} M \quad (59)$$

We adopt the former since no matrix inversions are involved and our application does not involve precise calculation which would be involved in an actual estimator.

The simulation also allows for any component of the state to represent a "consider" variable, that is one which the estimator does not attempt to improve. This can be done simply by changing the covariance of the consider variable back to the values which existed before the update.

2.2.5 Output of Program

With all the previous calculations, the simulation provides a time history of the trajectory with all of the attendant quantities such as gimbal angles and rates together with a time history of the entire state covariance matrix. Of more direct interest is the statistics of the error in position of one reflector (reflector i) with respect to another reflector (reflector j). If S_{ii} is the 3 x 3 submatrix giving the covariance of position components of reflector i and S_{ij} is the 3 x 3 submatrix of correlative terms relating reflectors i and j then the required covariance of position difference is given by

$$S_{ii} + S_{jj} - S_{ij} - S_{ji}$$

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An average strain error covariance for M reflectors can be fabricated by summing for pairs of reflectors. This is expressed as

$$S_{\text{avg}} = \frac{2}{M(M-1)} \sum_{i,j}^M (S_{ii} - S_{ij}) \quad (60)$$

Also obtainable from the state covariance matrix is an estimate of obtainable pointing error. The derivations of the computations for this are presented in Appendix B. We must note that this pointing error is based upon the covariance of the simulation under discussion and not necessarily the covariance expected to be provided by an actual on-board mechanization of the navigation accuracies derived from ranging.

2.2.6 Results

The covariance simulation which has been described was applied to a shuttle borne ranging device travelling in a 180 mm altitude orbit having a 55 degree inclination. The node was selected so that the first pass-travelled across a point 1 degree east of the central reflector of the array which was at 33 degrees of latitude. The outrigger reflectors were 200 km apart on a square with north-south, east-west sides. The spacecraft was travelling northeastward at this point. The node was reinitialized for a second pass over this same point but travelling southeastward. The ground tracks for these are shown in Figure 26. The measurement policy previously described was used. Figure 27 presents the gimbal angle history for the first pass. This suggests that a renumbering of some of the reflectors might reduce the maximum gimbal rate slightly but the rate requirements are not serious. Range history is presented in Figure 28.

Presentation of covariance results in terms of 3 x 3 matrices or just their diagonal terms can sometimes obscure the true nature of the accuracies obtainable because of the inappropriateness of the coordinate system selected. In order to better see the degree of accuracy present in the reflector relative position covariance, the eigenvalues of the covariance matrix were generated. Although time histories of these as the ranging process continues represents a principle axis coordinate system which is changing with time, the results of doing this still provides a better indication of system performance.

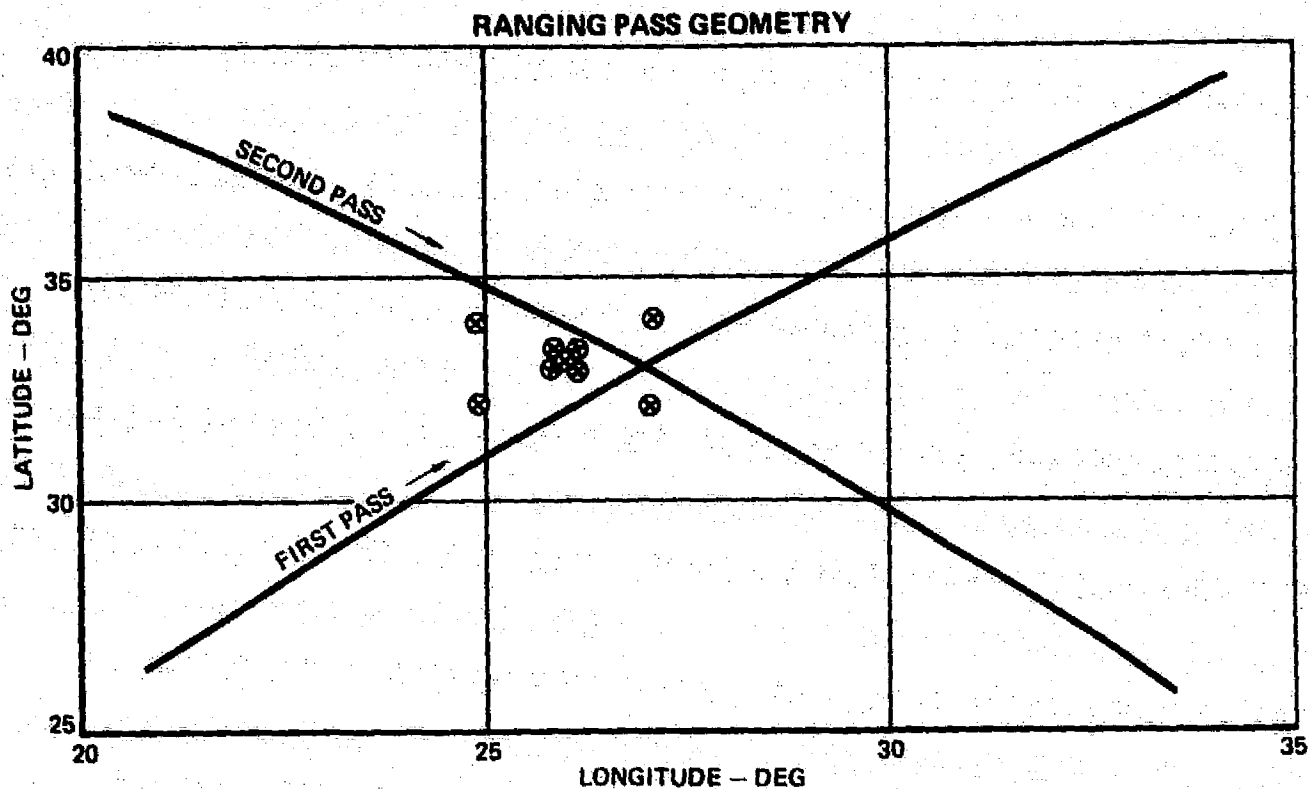


Figure 26

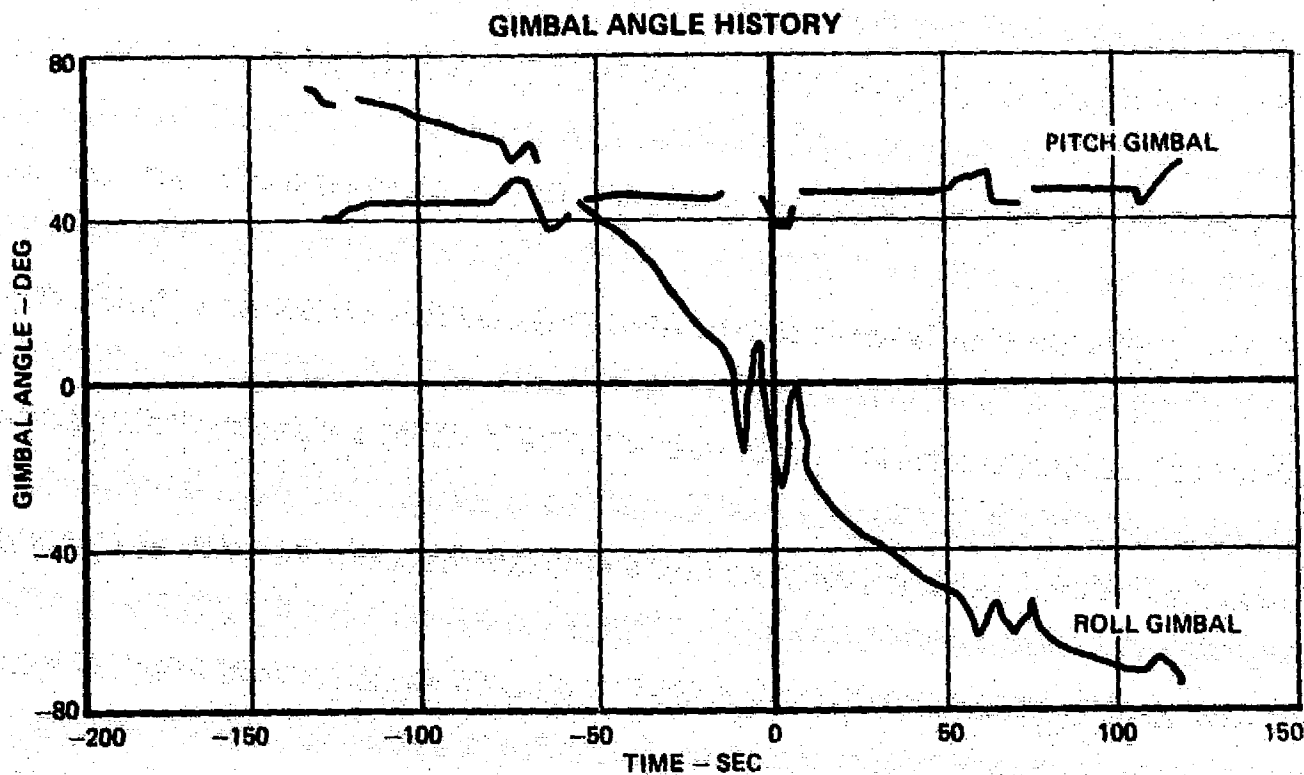


Figure 27

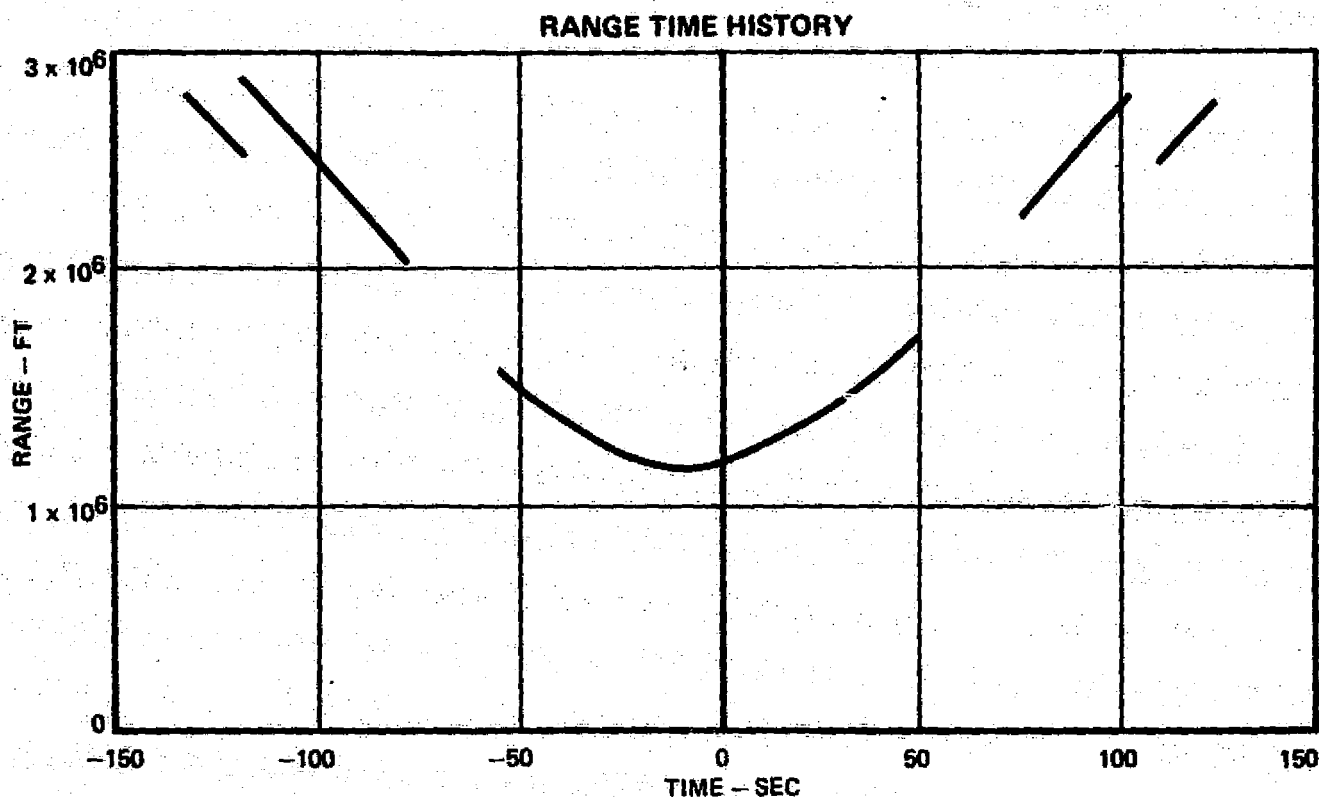


Figure 28

A cursory examination of the experimental procedure would lead one to the conclusion that the position difference accuracy between a pair of reflectors would be on the order of the random error of the ranging measurements and that both the range bias and the spacecraft position error should cancel in the differencing process. Indeed a one-dimensional model of the experiment with a stationary observer and one range measurement on each reflector will give a range difference variance of

$$\sigma^2 = \frac{2\sigma_T^2}{\sigma_T^2 + \sigma_m^2} \sigma_m^2 \quad (61)$$

where σ_T^2 is the a priori variance of reflector position. However, the simulation did not behave this way with target reflectors having an angular separation as viewed from the spacecraft. The range bias cancels but the spacecraft position error does not. Appendix C shows why. In this case the system behaves as if the random error variance is $\sigma_m^2 + (\sigma_{Ty}^2 + 2\sigma_{Sy}^2) \sin^2 \delta$ instead of just σ_m^2 where 2δ is the separation angle and σ_{Sy}^2 is the variance of spacecraft position

perpendicular to the line of sight and in the plane of the reflectors. Since σ_{Sy} can be quite large, $\sigma_{Sy} \sin \delta$ can easily dominate over σ_m .

Furthermore additional range measurements do not improve this since spacecraft position error is systematic during the observation period.

Figure 29 shows the principal 1 σ position difference accuracies for the relative positions of reflectors 1 and 2. These are the square roots of the eigenvalues. Notice that the minimum eigenvalue does not immediately decrease as the reflectors in question are viewed but must await some reduction in spacecraft position. The intermediate eigenvalue begins to be reduced when sufficient angular travel makes another direction observable. The position difference normal to the orbit plane requires a second pass which observes this component.

The spacecraft position determination which is involved in the first pass is shown in Figure 30. It seems apparent that a spacecraft accuracy of less than 1 ft can only briefly be obtained near closest approach. This is the limiting effect in experiment accuracy. Modification of the sampling policy cannot

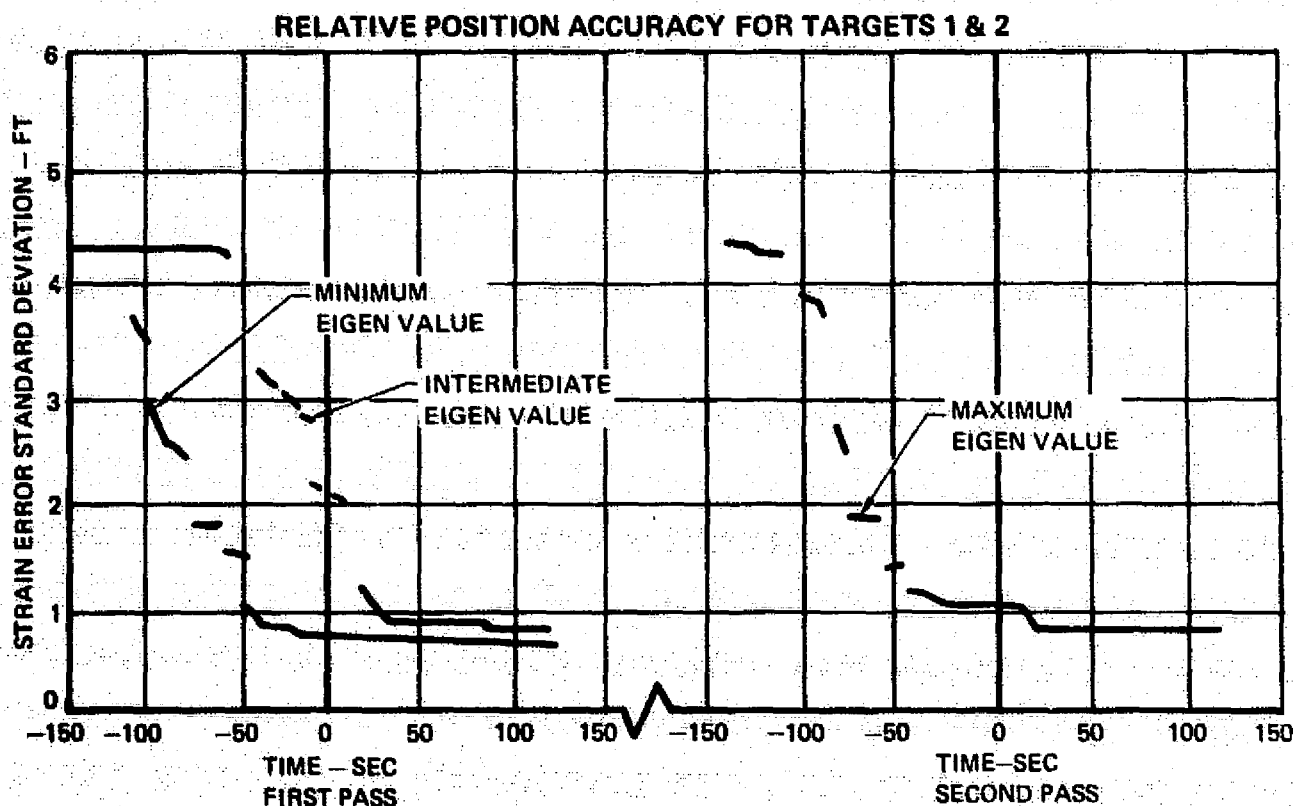


Figure 29

appreciably improve this with the possible exception of additional independent tracking of the spacecraft. Any additional orbit determination must measure to an accuracy of greater than 1 ft referenced to the reflector grid. This is the rationale for the extra "outrigger" reflectors positioned so as to provide good triangulation for spacecraft position.

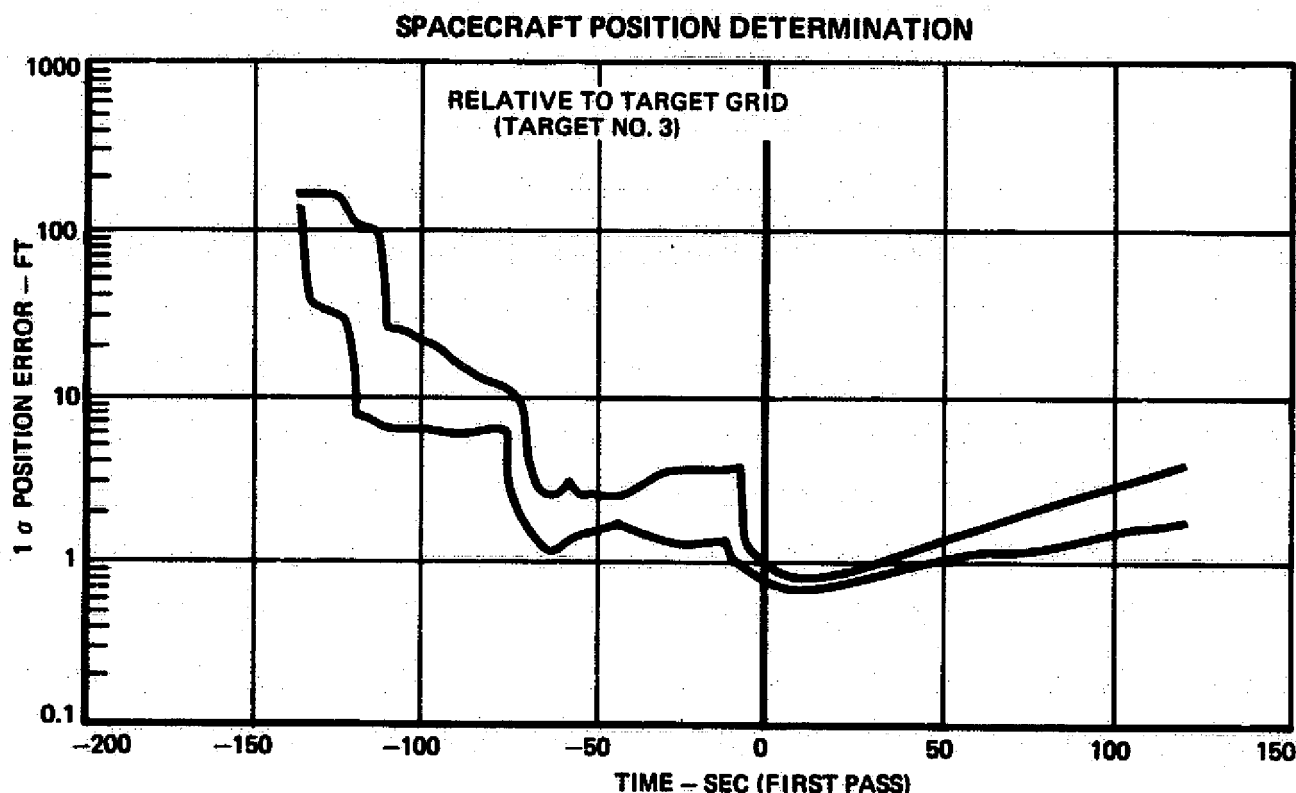


Figure 30

The effect of moving the outrigger reflectors closer to the target grid was investigated. Placement of these four reflectors at 100,000 meters rather than at 200,000 m degrades the strain accuracies in the target grid by about 30%. The outrigger positions have been subsequently changed to be at 200,000 m but situated north-south and east-west of the target grid. This is to provide more lateral viewing of the spacecraft because of its 55° inclined orbit over the area.

The pointing uncertainty due to the uncertainties remaining in the relative spacecraft reflector positions is shown in Figure 31.

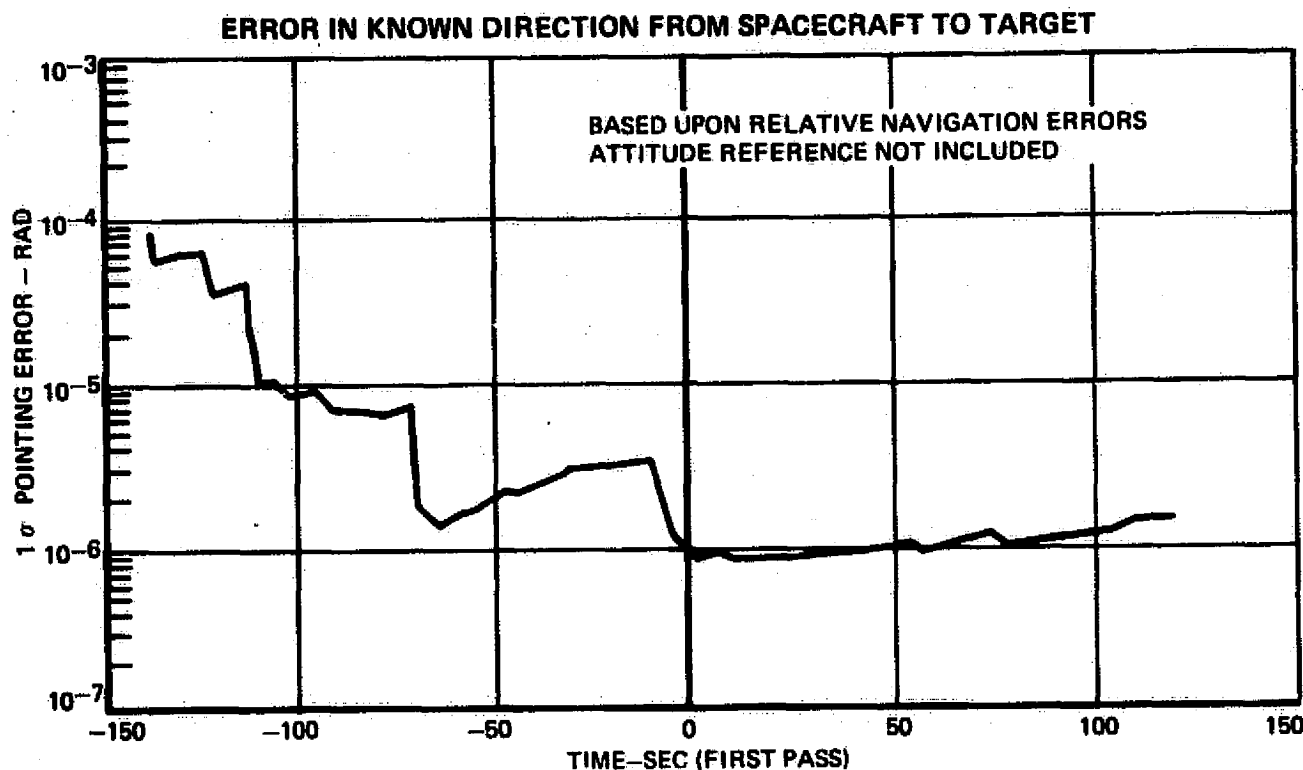


Figure 31

The simulation was exercised to determine the changes which would result from more remote encounters of the spacecraft with the reflector grid. This was done by varying the orbit node so that orbit ground tracks would have specified longitude differences with the reflector grid of the grid latitude. This was done mainly to check ranges and gimbal angle histories to be expected. Figure 32 shows that the accuracy deteriorates rapidly after 4 degrees of separation. Figure 33 shows the gimbal rate histories for a longitude separation of 6 degrees. Further mission analysis studies are not within the jurisdiction of the contract. All that was needed was corroboration of the selection pointing system features from an experiment accuracy point of view. The results so far indicate that the pointing system design is compatible with experiment objectives.

2.3 Acquisition Process Analysis

The expected angular uncertainties due to ephemeris and target location errors significantly exceed the pointing uncertainties due to attitude reference and pointing control errors, at least until the first Kalman filter update is accomplished. The acquisition process consists of a sequential search over the region

RECONSTRUCTION ERROR ACHIEVABLE AS A FUNCTION OF DISTANCE FROM GRID CENTER

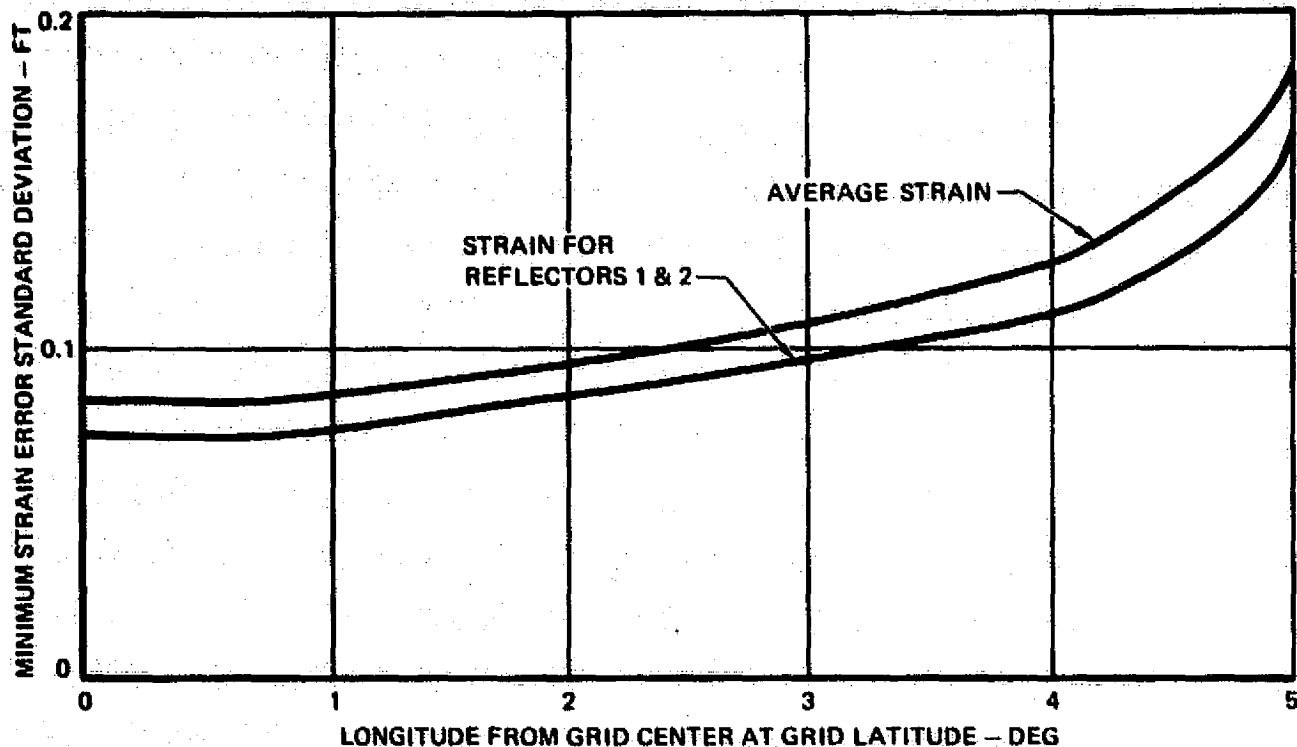


Figure 32

GIMBAL ANGLE HISTORY

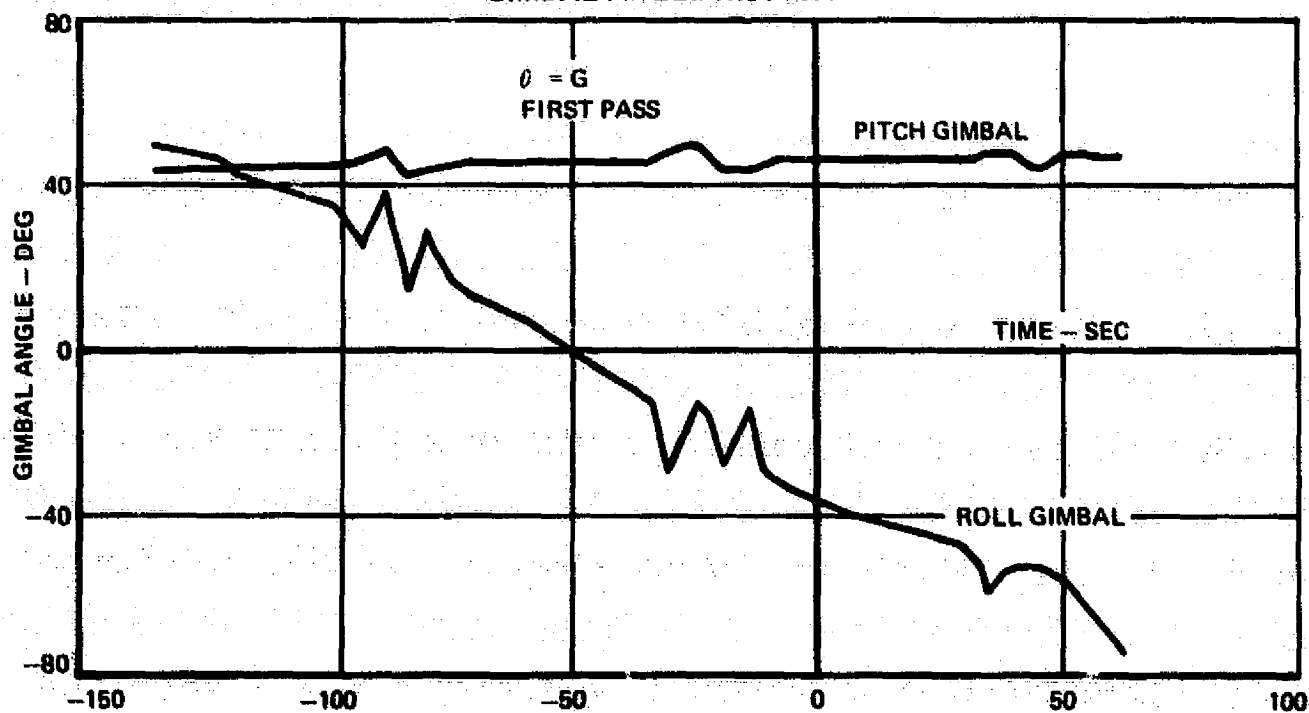


Figure 33

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of uncertainty until a valid target is detected. At that point, the angular uncertainties are reduced, and the normal ranging mode commences.

Let $f_0(\bar{X})$ be the apriori probability distribution of target location, where $\bar{X}$ is a two dimensional angular vector. Let $g(\bar{X} - \bar{X}_n)$ be the probability of detection, given the target is at $\bar{X}$ and the beam center is at $\bar{X}_n$ (for nth trial). Then the probability of detection by the nth trial, $P_D(n)$, is,

$$P_D(n) = 1 - \int_{-\infty}^{\infty} f_0(\bar{X}) \prod_{i=1}^n [1 - g(\bar{X} - \bar{X}_i)] d\bar{X} \quad (62)$$

The probability of detection on the nth trial, $P_d(n)$, is simply,

$$P_d(n) = P_D(n) - P_D(n-1) = \int_{-\infty}^{\infty} f_0(\bar{X}) \prod_{i=1}^{n-1} [1 - g(\bar{X} - \bar{X}_i)] g(\bar{X} - \bar{X}_n) d\bar{X} \quad (63)$$

The mean time to acquire is,

$$T = \Delta T \sum_{i=1}^{\infty} i P_d(i) \quad (64)$$

If we choose the set of pointing locations ($\bar{X}_i$, $i=1$ to n) to maximize $P_D(n)$, we would have n simultaneous equations to satisfy (for $K = 1$ to n),

$$\int_{-\infty}^{\infty} f_0(\bar{X}) \prod_{\substack{i=1 \\ i \neq k}}^n [1 - g(\bar{X} - \bar{X}_i)] g^1(\bar{X} - \bar{X}_k) d\bar{X} = 0 \quad (65)$$

The values which satisfy these equations could then be ordered to minimize T , given a detection occurred in the n trials.

These are only two of the possible cost functions. Whatever cost function we choose, one additional factor should be considered. We choose a cost function such that a sequence $\bar{X}_i$, $i=1$ to n is optimal for n trials. Then, if the target is not detected, we may choose to extend the search for another m trials. Then, if the sequence $\bar{X}_i$, $i=1$ to n previously defined is contained in the sequence $\bar{X}_i$, $i=1$ to $n+m$ which is optimal for $n+m$ trials, the sequence is uniformly optimal.

One such uniformly optimal sequence is obtained by simply choosing $\bar{X}_n$ to maximize $p_d(n)$, given the $n-1$ previous trials. The sequence is obtained by choosing $\bar{X}_1$, then $\bar{X}_2$, etc. The sequence so defined is not necessarily optimum in terms of minimum mean time to detect the target or maximizing the probability of detection in n trials, but it is uniformly optimal.

The optimal sequence is difficult to define, since we do not know $g(\bar{X})$. This results from the fact that $g(\bar{X})$, the detection probability, is significantly affected by atmospheric scintillation. This affect can be modeled by assuming the variance of the log of the detected signal (C) is constant, but unknown, and that $h(C)$ is the probability density of C . Thus, for our optimal sequence, $\bar{X}_n$ would be chosen to maximize,

$$p_d(n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(\bar{X}) \prod_{i=1}^{n-1} [1 - g(\bar{X} - \bar{X}_i | C)] g(\bar{X} - \bar{X}_n | C) h(C) d\bar{X} dC \quad (66)$$

One more problem remains to be considered. Consider the possibility that a cloud was obscuring the target for exactly the first K trials, and let $q(K)$ be the probability of this event. Then,

$$p_d(n) = \sum_{K=0}^{n-1} q(K) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(\bar{X}) \prod_{i=K+1}^{n-1} [1 - g(\bar{X} - \bar{X}_i | C)] H(C) d\bar{X} dC \quad (67)$$

Equation (67) must be solved for the $\bar{X}_n$ that maximizes $p_d(n)$ for $n = 1$ to some relatively large number (perhaps up to 200), in order to obtain an optimal acquisition search sequence. Then, since the models for $q(K)$ and $h(C)$ are not well known, it would probably be desirable to alter the parameters and generate a new optimal sequence, and compare the sequences thus generated. Finally, $g(\bar{X}|C)$ is not stationary with time since the range is decreasing rather rapidly when acquisition begins. The rate of decrease of range, and hence rate of increase in mean signal level, is a function of the distance of the target from the Shuttle orbit plane, thus to be as precise as possible, a new optimal sequence should be generated for each target pass.

One minor problem, of course, is that $g(\bar{X}|C)$ cannot be described in closed form when log-normal scintillation is considered. Thus, there is simply no way to

evaluate Equation 67 except numerically, and an enormous computational load is anticipated even to generate a short optimal sequence.

A more practical approach needs to be devised in order to obtain some basic design data. First, consider the case where the scintillation variance is small and the mean signal large. The detection probability will then be nearly 1 in the central portion of the beam, and nearly zero outside of the central region. Thus, we could approximate,

$$\begin{aligned} g(\bar{X}) &= 1, \quad |\bar{X}| < \rho \\ &= 0, \quad |\bar{X}| > \rho \end{aligned}$$

If we arrange a minimum length search sequence so that all possible target locations within a specified search radius are within at least one detection radius (ρ), the resulting set of search locations, $\bar{X}_n$, would be describable as a two dimensional sampling grid with any three adjacent points at the corners of an equilateral triangle of dimension $\rho\sqrt{3}$ on a side. The overall detection probability would be simply the probability that the target was within the region searched, assuming no obscuration. This sampling grid structure was chosen for the balance of the acquisition studies; the sample spacing was considered a variable.

The mean return signal S_0 is a function of the planar distance, v , between the beam center and the target, and the e^{-1} beam radius, r_0 .

$$S_0 = \frac{a}{r_0^2} e^{-r^2/r_0^2} \quad (68)$$

where a = constant determined by the link conditions.

When log-normal scintillation is postulated, the observed signal energy, S , is a random variable with a probability distribution,

$$f(s) = \frac{1}{2S\sqrt{2\pi C}} \exp \left\{ -\left[\frac{1}{2} \ln \left(\frac{S}{S_0} \right) + C \right]^2 / 2C \right\} \quad (69)$$

where C = variance of the log of S

Next, assume that the receiver employs an ideal photo-electron counting, sliding window integrator. Then, when the signal is wholly within the window, the

probability of detection given S, for a threshold, L, is,

$$P_D = 1 - \sum_{K=0}^{L-1} \frac{(\eta S)^K}{K!} e^{-\eta S} \quad (70)$$

where S is the signal energy in photons/pulse
and η = quantum efficiency of the detector

Thus, the overall probability of detection is simply,

$$P_D = 1 - \int_0^{\infty} \sum_{K=0}^{L-1} \frac{(\eta S)^K}{K!} e^{-\eta S} f(S) dS \quad (71)$$

This integral was computed numerically for the limiting case, $C = 0.6$, for typical threshold levels, yielding the curves in Figure 34. The average signal is simply ηS_0 , as defined previously. An empirical expression was derived for these curves, of the form,

$$P_D \approx 1 - \exp \left(- \sum_{K=0}^{10} a_K (\eta S_0)^K \right) \quad (72)$$

where the a_K were computed numerically for each threshold setting.

Next, let the points in the sampling grid be defined as X_{ij} , Y_j ,

$$X_{ij} = id + \frac{d}{2} (j \bmod 2) \quad (73)$$

$$Y_j = jd + \frac{\sqrt{3}}{2} d \quad (74)$$

and

$$X_{ij}^2 + Y_j^2 \leq R^2$$

where R is the desired search radius.

Next, assume that the target is located at a point, X_T , Y_T . Now, for any particular pulse pointed at the i,j sample grid location, the distance between the beam center and the target is defined by,

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PROBABILITY OF DETECTION VS AVERAGE SIGNAL
(WORST CASE ATMOSPHERICS)

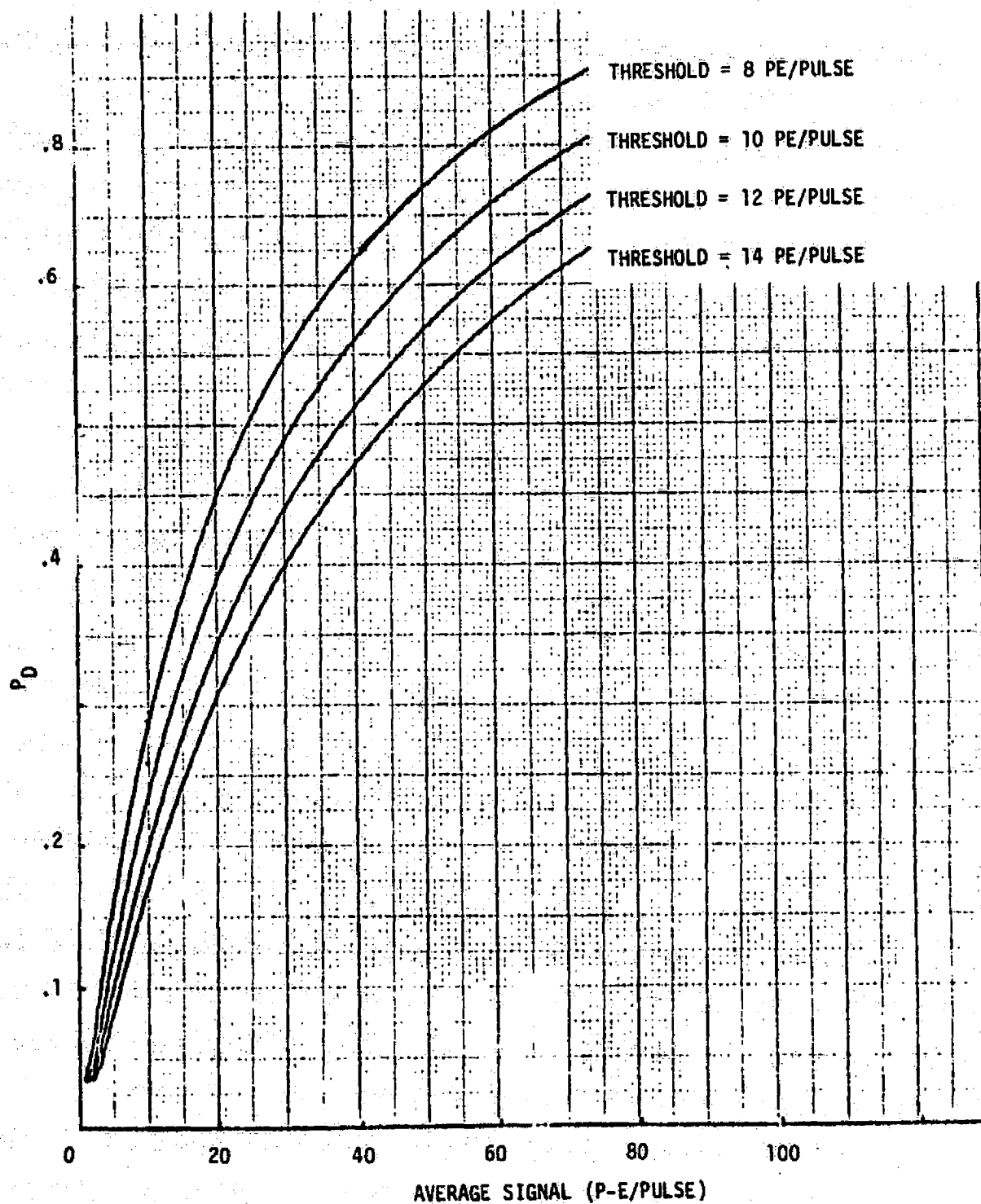


Figure 34

$$r_{ij}^2 = (x_{ij} - x_T)^2 + (y_j - y_T)^2 \quad (75)$$

The average signal $(\eta S_o)_{ij}$ expected is,

$$(\eta S_o)_{ij} = \frac{a}{r_o^2} \exp - \frac{r_{ij}^2}{r_o^2} \quad (76)$$

The probability of detection in one complete pass through the sampling grid is simply,

$$P_D = 1 - \sum_{i,j} \exp \left(- \sum_{K=0}^{10} A_K (\eta S_{oi,j})^K \right) \quad (77)$$

The beam radius (r_o) has a significant effect on the detection probability. If the radius is either very large or very small, the detection probability is small, thus we conclude that an optimum beam radius can be defined for any specific set of conditions. Figure 35 shows the optimum beam radius as a function of the number of samples in the grid and the search region radius, where the nominal received signal, from a target on beam center, was 100 p-e/pulse, with a 0.5 mrad transmit beam width.

The detection threshold affects the optimum value of r_o , as shown in Figure 36 resulting in a decrease of the optimum beam radius as the threshold is increased.

These calculations were performed under the presumption that the target was near the center of the search region. Figure 37 shows the effect of moving the target various distances from the center of the search region on the probability of missed detection. The effect of varying the threshold on the probability of missed detection is shown in Figure 38. For both of these figures, the target was located on a radial inclined 30° from the horizontal axis of the sampling grid. We found that no significant changes resulted from varying the angle of the radial.

The next step was to assume that the probability distribution of the target location was a zero mean circular normal distribution. Thus, the probability that the target is at a distance r_T from the center of the sampling grid is,

$$p(r_T) = \frac{r_T}{\sigma^2} \exp (-r_T^2/2\sigma^2) \quad (78)$$

where,

$\sigma_x = \sigma_y = \sigma$ = standard deviation of the location error

OPTIMUM BEAM RADIUS VS. NUMBER OF SAMPLES IN GRID FOR 10 PE/PULSE DETECTION THRESHOLD

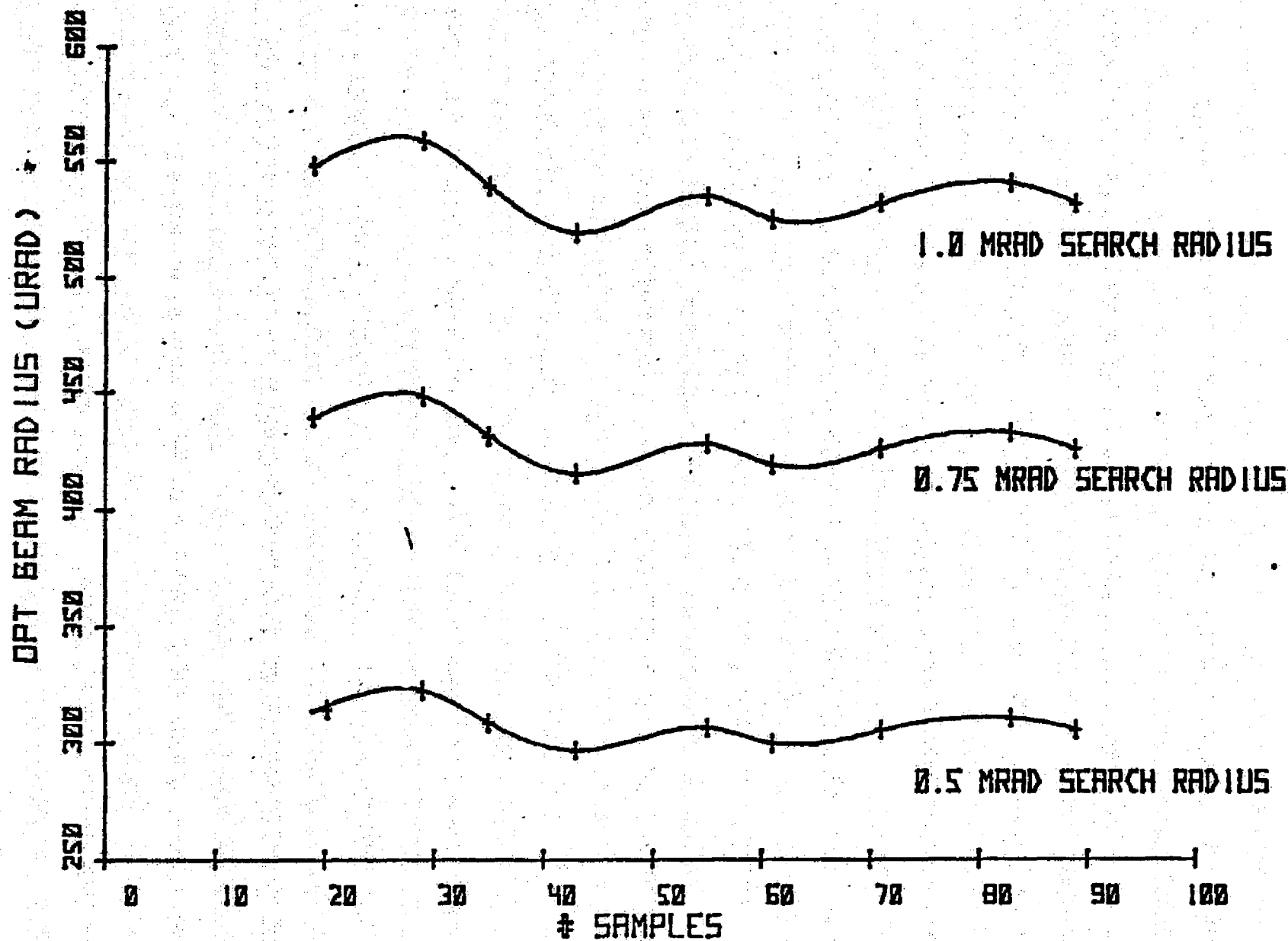


Figure 35

EFFECT OF THRESHOLD ON OPTIMAL BEAM RADIUS

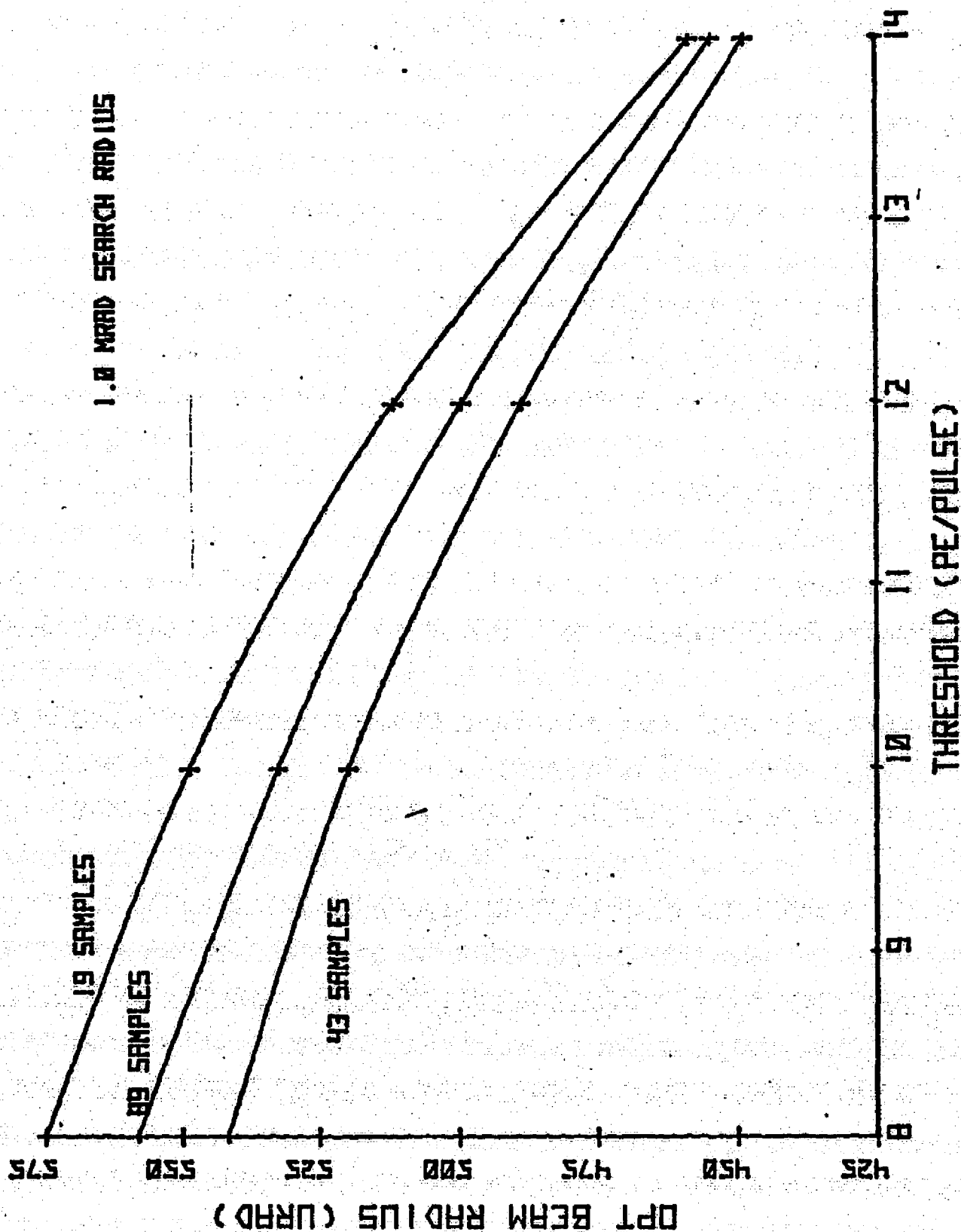


Figure 36

PROBABILITY OF MISS-DETECTION VS. RADIAL DISTANCE OF TARGET FROM CENTER OF SAMPLING GRID

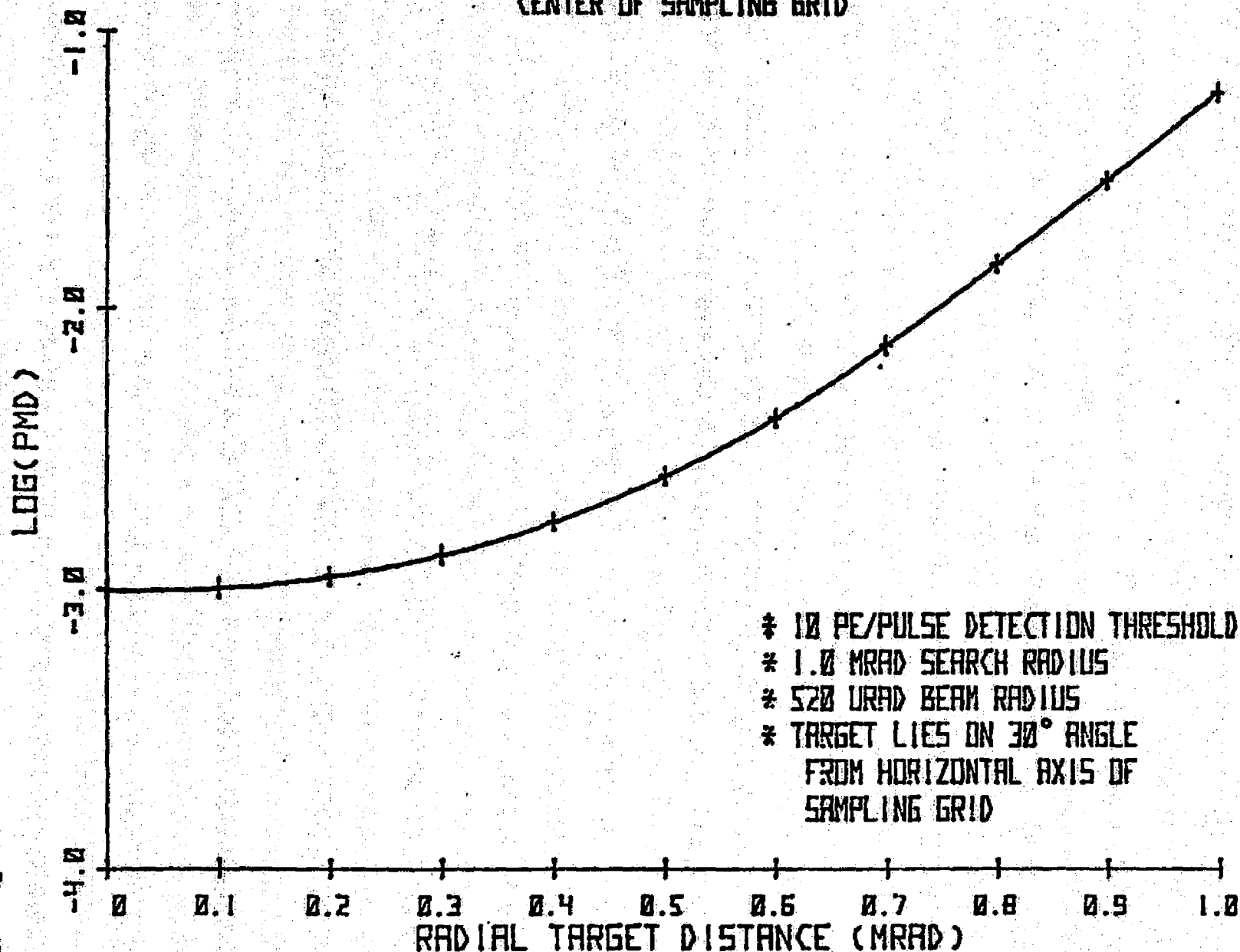


Figure 37

EFFECT OF THRESHOLD ON PROBABILITY OF MISS-DETECTION

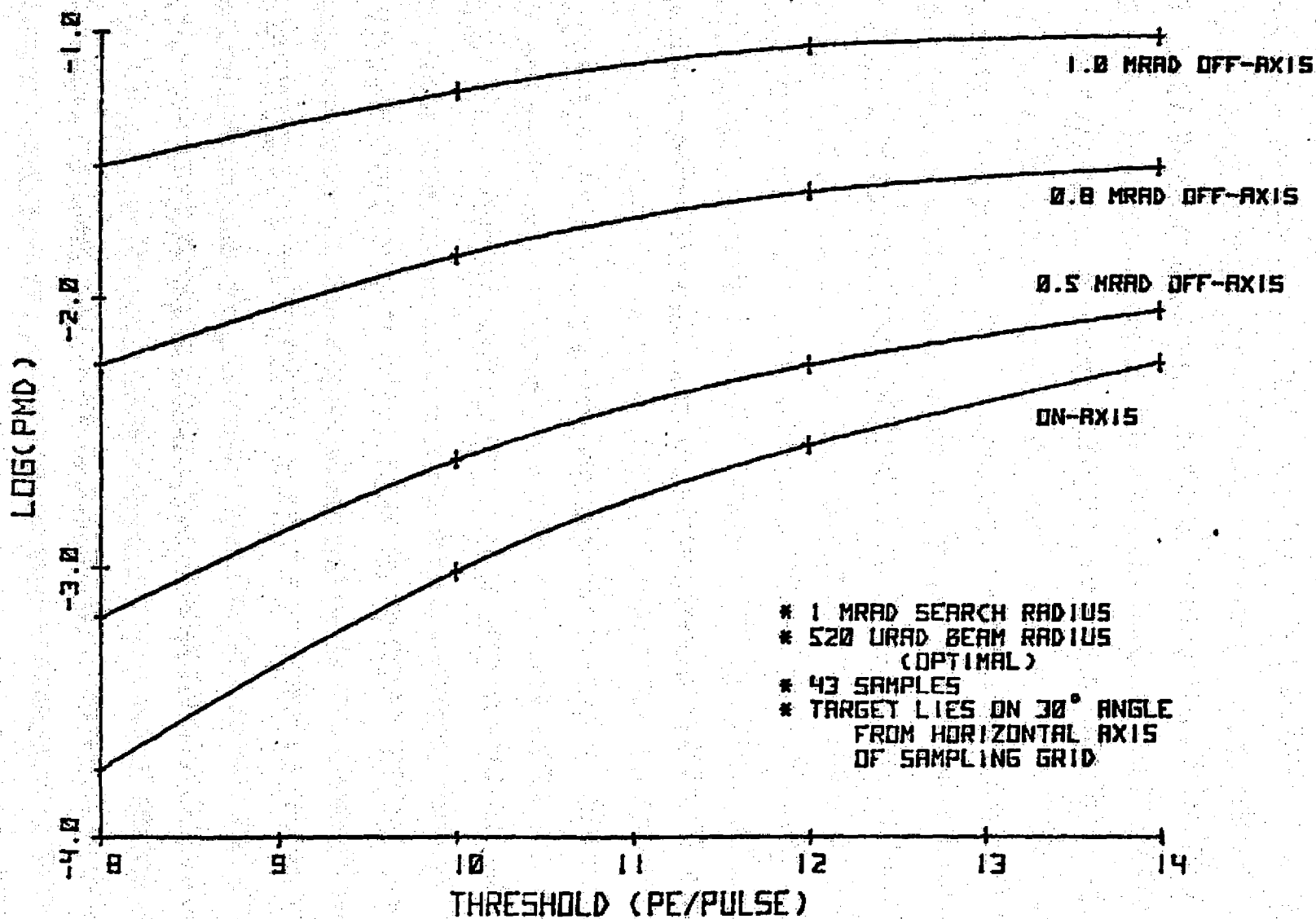


Figure 38

A conservative value of $\sigma = 0.2$ mrad was assumed, and the overall probability of missed detection was computed as a function of the number of samples in the grid, as shown in Figure 39.

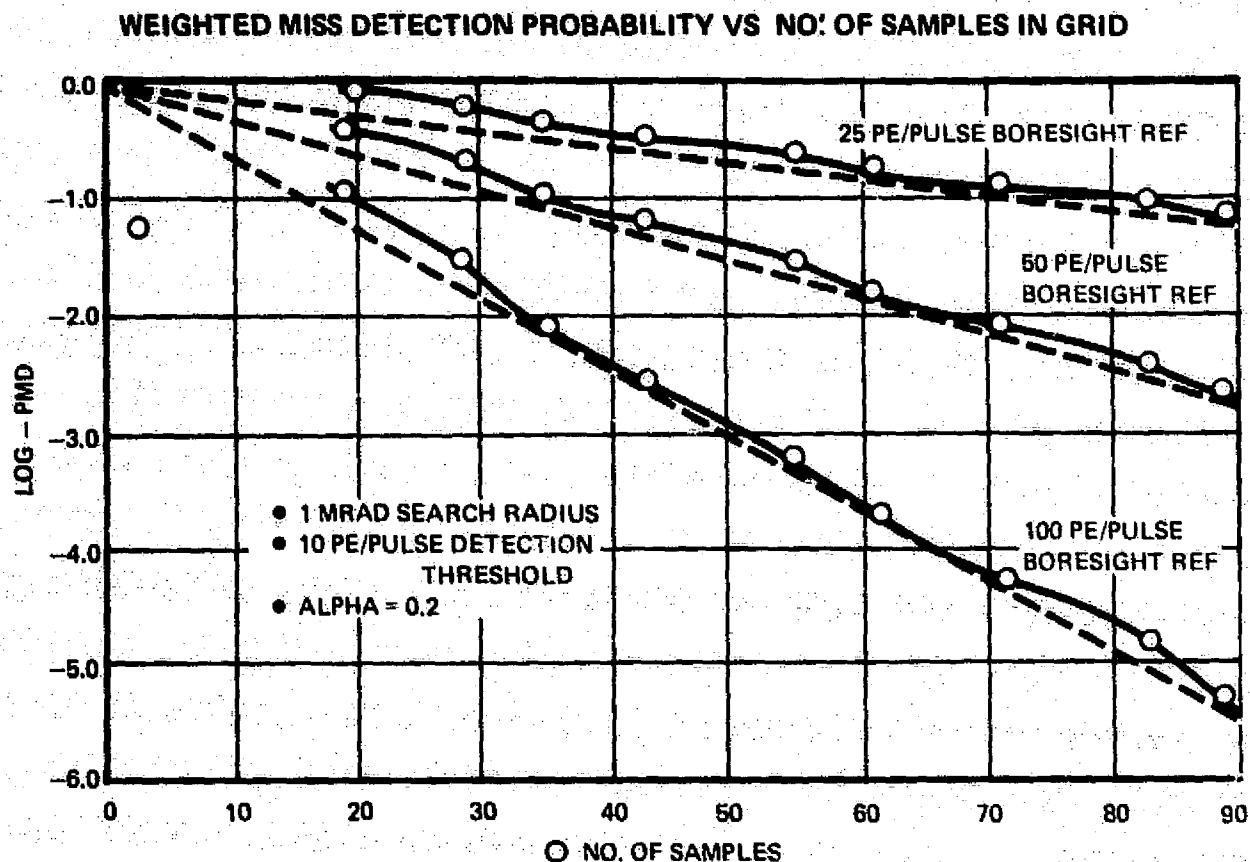


Figure 39

Also shown in Figure 39 are three phantom lines, drawn through the origin and the lowest points on the computed curves. These lines represent constant payoff lines, i.e., the log of probability of missed detection is linearly proportional to the number of trials. The significance is that the probability of missed detection in N trials is the same whether the number of trials represents maximally interlaced samples in the grid or repeated trials of a smaller sized grid. Thus, for the most robust link shown, there is no practical reason to define a sampling grid larger than ~ 35 samples. However, for less robust links, larger sampling grids represent additional improvement in detection probability. The slopes of the bounding lines appear to be linearly proportional to nominal signal strength. Thus, we can approximate, for this specific set of conditions,

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$$P_{MD} \approx 10^{-.0006EN} \quad (79)$$

where N is the number of pulses and E is the expected number of photo-electrons per pulse from a target on beam center where the beam divergence is 0.5 mrad.

The validity of this approximate expression can be shown in the following manner. Consider the case where the target is located in the center of the sampling grid. The sampling grid can be approximated as a set of sampling points located on concentric circles about the target. The spacing between adjacent circles is Δr . The number of sample points on the i th circle from the center is approximately $6i$. Assume there are n circles, extending to $R = n\Delta r$ in radial distance, with a total number of samples, $N = 1 + 3n(n+1)$. The mean signal strength for a sample on the i th circle is,

$$S_{o,i} = \frac{A}{r_o^2} e^{-(i\Delta r/r_o)^2} \quad (80)$$

As noted previously, the probability of detection for this sample is,

$$P_{D_i} \approx 1 - \exp \left(- \sum_{K=1}^{10} A_K S_{o,i}^K \right)$$

Thus, the log of the probability of missed detection for the $6i$ trials on the i th circle is,

$$P_i = -6i \sum_{K=1}^{10} A_K \left(\frac{A}{r_o^2} \right)^K e^{(-K(i\Delta r/r_o)^2)} \quad (81)$$

Then,

$$\begin{aligned} \ln P_{MD} &= \sum_{i=1}^n P_i = -6 \sum_{K=1}^{10} A_K \left(\frac{A}{r_o^2} \right)^K \sum_{i=1}^n i e^{-\left(\frac{K\Delta r^2}{r_o^2} \right) i^2} \\ &\approx -6 \sum_{K=1}^{10} A_K \left(\frac{A}{r_o^2} \right)^K \int_0^{n+1} i e^{-\left(\frac{K\Delta r^2}{r_o^2} \right) i^2} di \end{aligned}$$

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$$\begin{aligned}
 & \sum_{K=1}^{10} A_K \left(\frac{A}{r_o^2} \right)^K \left(\frac{r_o^2}{K \Delta r^2} \right) \left(1 - e^{-(n+1)^2 K \Delta r^2 / r_o^2} \right) \\
 & \approx - \frac{3}{\Delta r^2} \sum_{K=1}^{10} A_K \frac{A^K}{r_o^{2K-2}}
 \end{aligned} \tag{82}$$

Since $\Delta r = R/n$, and $n \approx \sqrt{N/3}$,

$$\ln P_{MD} \approx - \frac{N}{R^2} \left(A_1 A + A_2 \frac{A^2}{r_o^2} + \dots \right) \tag{83}$$

Thus, to a first order approximation, the log of the probability of missed detection is linearly proportional to both N and signal energy per pulse, and inversely proportional to the acquisition search area. As long as the search radius is considerably larger than the standard deviation of the target location uncertainty, and the target grid spacing is small compared to the nominal beam radius, this approximation should be quite reasonable regardless of target location within the grid, as noted during the numerical analysis. Furthermore, note that r_o , the beam radius for acquisition, does not appear as a first order effect on the missed detection probability, provided that r_o is large enough that the second and higher order terms do not effect the results.

Thus, we conclude that for reasonable system parameters, acquisition is likely to be quite reliable even for worst case atmospheric scintillation. Further, note that at nominal acquisition conditions, i.e., 20° elevation, the range is reducing at $\sim 7.5\text{km/sec}$, thus the mean return signal energy will increase almost 40% in the first 10 seconds of the acquisition process, further enhancing the acquisition probability. Also, we note that the optimum beam radius for acquisition is sufficiently large that the second and higher ordered terms in the approximation are negligible. Thus, we find that choosing a beam radius somewhat larger than the calculated optimum yields better results than choosing a smaller beam radius (the A_K are alternating in sign). Finally, we found that the controlling parameters were the nominal energy per pulse and the grid sample density (R^2/N).

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2.4 Stable Platforms

A number of experiment pointing systems have been proposed for Shuttle/Spacelab applications. The general thrust of these pointing systems is to provide a general purpose mount which is capable of pointing experiments with varying levels of accuracy and stability. Four of these systems were evaluated for compatibility with a laser ranging experiment. The initial screening resulted in concluding that only one of the proposed pointing systems was potentially suitable for a laser ranging experiment. The four pointing systems considered were the Instrument Pointing System (IPS), the Miniature Pointing Mount (MPM), the Small Instrument Pointing System (SIPS), and the Annular Suspension and Pointing System (ASPS).

Both the IPS and the MPM employ mechanical isolators to reduce the sensitivity of the pointing system to Shuttle disturbances. These isolators allow the gimbal centroid to move with respect to the Shuttle structure. Any unknown motion of the gimbal centroid with respect to the Shuttle coordinate system results in a residual translation error of the ranging data. Consequently, both the IPS and the MPM were considered basically unsuitable for a precision laser ranging experiment.

The ASPS concept includes a magnetically suspended fine pointing gimbal system inside a conventional coarse gimbal system. This concept was not considered suitable for applications demanding significant angular agility.

The remaining pointing system, SIPS, is considered basically suitable for a laser ranging experiment; no fundamental incompatibilities were identified, although the design slew rates and angular acceleration capabilities will limit the experiment capability to make ranging measurements to target grids composed of many and/or widely spaced targets. A number of areas of concern were identified, as discussed in the following paragraphs.

The first area of concern is one of safety. This concern is due to the design of the SIPS gimbals, which allow full hemispherical pointing capability. Since the laser ranging system employs a high peak power laser transmitter, a system of interlocks and control procedures will be required to ensure that no significant amount of laser energy is incident on any part of the Shuttle, and spectrally selective window covers are recommended for all windows in the Shuttle and Spacelab which could be, or could view any part of the Shuttle illuminated by the laser ranging system transmitter.

The second area of concern results from the requirement to transfer the range measurement to the Shuttle center of gravity (cg). This transfer requires knowledge of the SIPS gimbal angles and position of the gimbal centroids in Shuttle coordinates. As discussed in Section 2.2, the location of the Shuttle cg is included in the mathematical model of the system. However, any uncertainty in gimbal angles or gimbal position results in a coupled ranging error. As currently implemented, the SIPS gimbal angles are sensed with 12 bit precision shaft encoders (~ 1.5 mrad resolution).

The distance from the gimbal centroid to the Shuttle cg is a function of the location of the experiment within the payload bay, and can range from 2.5 to 12 meters; the maximum ranging error coupled into the measurement by gimbal angle readout resolution is on the order of $1.5 \times 10^{-3} \times D/\sqrt{12} \approx 0.5$ cm (rms). Note that angular deflections due the thermal gradients and/or structural flexures could add directly to this uncertainty, but for the purposes of this study were assumed to be an order of magnitude smaller than the readout resolution.

The third area of concern is the relationship between Shuttle attitude, during the pass, and the required angular accelerations about the gimbal axes to keep the pointing vector oriented toward the target. This is a concern because of the limited angular travel of both the inner (right-left) gimbal and the elevation gimbal. The Shuttle orientation strategy devised in the mission simulation (Section 2.2) resulted in selecting a modified, x-axis perpendicular to orbit plane, (POP) inertial attitude such that the Shuttle Z axis intercepted the target grid center at the point of closest approach. The target grid, under this condition, was located within a few degrees of the Shuttle Y-Z plane throughout the ranging pass. This is not an optimum attitude strategy for the SIPS mounted laser ranging experiment, since either an azimuth gimbal flip is required, or the SIPS elevation axis design would have to be modified to permit a 180° elevation capability. The simplest solution seems to be to tilt the Shuttle Z axis by an angle, i , from the grid center at the point of closest approach (retaining the Y axis in the orbit plane). Then, the maximum angular acceleration required about the azimuth axis is approximately $.02/\sin^2$ (degrees/sec²), for a Shuttle altitude of ~ 330 km (180 nmi). The maximum azimuth gimbal acceleration capability is assumed to be $0.2^\circ/\text{second}^2$, thus a tilt angle, i , of at least 18.5° is needed to allow continuous tracking at worst case conditions. It is somewhat undesirable to choose considerably larger tilt angles,

since this would bring the laser beam closer to the Shuttle or Spacelab structure, which is undesirable from a safety standpoint.

The fourth area of concern is the interface between the laser ranging experiment and the SIPS computer. As currently defined, the experiment will derive permitting error signals in up-down, and left-right coordinates which are forwarded to the SIPS computer to initiate corrective action. The concern is related to the effect of lags caused by data transfer delays and finite computer cycle times on the pointing error. Consider the following scenario. The experiment computer performs a number of essential functions, including a navigation function and an attitude reference function. The navigation function is included so that ranging data, after initial target acquisition, can be used to update the spacecraft position data and reduce the net pointing uncertainty for the balance of the ranging pass. The navigation data is used to predict the proper pointing direction and to position the range gate for the next scheduled transmit pulse. We anticipate this prediction will include the target range vector and the range rate vector, in stellar inertial coordinates. The attitude reference function maintains a current estimate of the actual pointing direction of the laser ranging experiment. The pointing command function is derived from the current pointing direction and the desired pointing direction for the next transmit pulse time. A prediction algorithm generates a series of pointing commands which will result in being at the correct pointing direction at the time of the next scheduled transmit pulse, with the correct angular velocity to maintain track during the pulse propagation time. This time period varies from two to six milliseconds, and the net change of direction during this interval is $2 V_T/C$, also referred to as the point ahead angle. At the point of closest approach, the point-ahead angle is about $55 \mu\text{rad}$, thus the maximum rotation rate of the pointing vector is less than $30 \mu\text{rad/ms}$. If the SIPS computer cannot accept and process a pointing error command in a prompt and precisely predictable manner, a dynamic error is introduced, which is proportional to the variation of the execution time and the pointing vector rotation rate.

The areas of concern discussed above are not insuperable problems; we conclude that a laser ranging experiment can be configured using the SIPS to point the transmit and receive telescopes. Integration of the laser ranging system and the SIPS, however, is considerably more complex than envisioned for the normal instrument payload for a SIPS application.

3.0 Experiment Preliminary Design

3.1 Pallet Mounted Experiment

The configuration of the pallet mounted experiment is illustrated in Figure 40. The experiment hardware is composed of two groups, the support electronics, mounted on coldplates attached to the pallet side, and the optical bench and associated components, which are mounted in a protective enclosure on a sill level platform. A removable cover is provided to shield the optical components from contamination and to maintain thermal stability when the experiment is not operating.

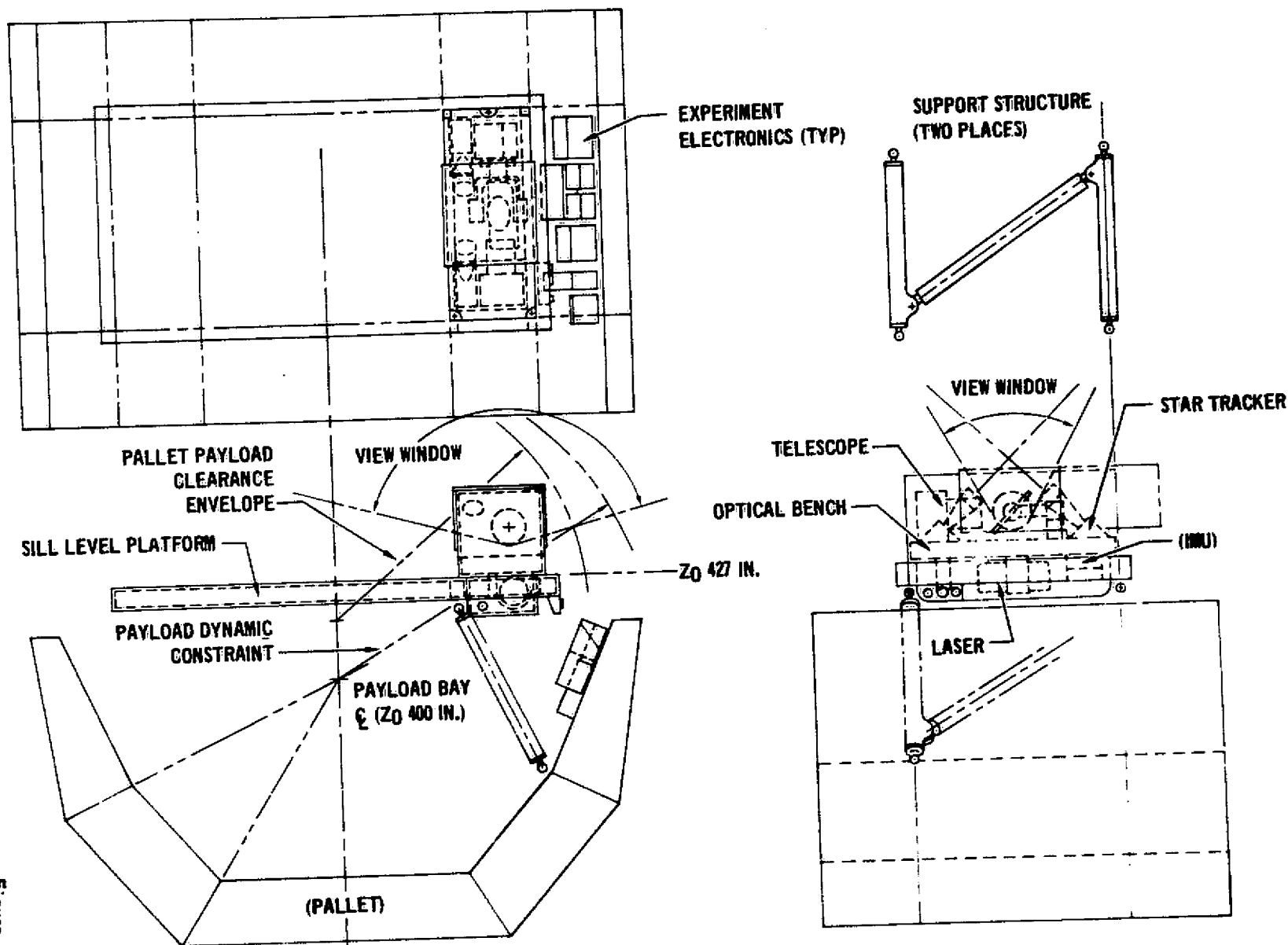
The optical bench mounted components include the laser transmitter, the optical receiver, star trackers, the attitude reference gyros, and the telescope and beam steering mirror.

The active elements of the system which are mounted on the optical bench are thermally insulated from the bench; energy which is dissipated within these components is discharged through an active thermoelectric system to the pallet supplied liquid loop. The enclosure is thermally insulated and equipped with heaters to maintain a stable thermal environment.

The support electronics components are mounted on coldplates connected to the pallet supplied liquid cooling system. If required, a thermoelectric mounting plate can be inserted between the component baseplate and the coldplate, allowing a precise bidirectional heat flow control to maintain a narrow temperature band within the component.

Figure 41 pictorially describes a typical ranging experiment. Approximately 45 minutes prior to the beginning of the ranging pass, the Shuttle is maneuvered to the nominal inertial orientation for the pass, and the experiment attitude reference system is initialized and calibrated during the next 15 to 20 minutes. The Shuttle orientation is nominally X-axis perpendicular to orbit plane (X-POP), with the Z-axis pointed directly at the center of the target grid at the point of closest approach. The Shuttle angular rates are controlled to as low as possible to minimize the need for thruster activity during the actual ranging operation. When the first target comes into view ($\geq 20^\circ$ elevation at the ground target), an acquisition sequence is initiated. A short target verify period

LASER RANGING EXPERIMENT - PALLET ACCOMODATION LAYOUT



3-2

Figure 40

follows the initial target detection. The experiment then enters the nominal ranging mode, which continues until the last target is no longer in view. At this time, the experiment is shut down and normal Shuttle activities resume.

LASER RANGING EXPERIMENT PICTORIAL

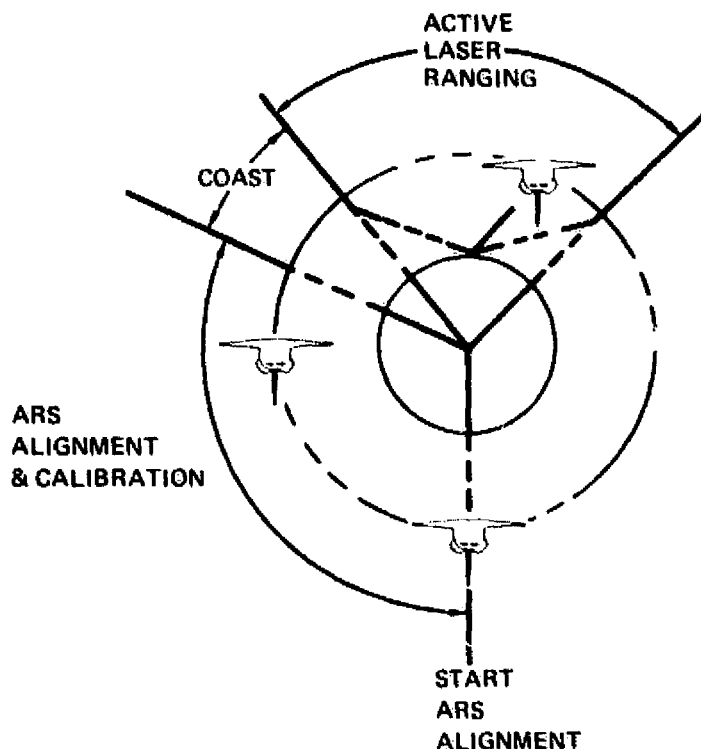


Figure 41

The data accumulated during the ranging pass is recorded for subsequent transmission to the ground. A typical individual measurement record includes both the transmit pulse departure time and the time of arrival of the reflected signal, attitude and gimbal angle data, a target label, the current Shuttle position data, and miscellaneous system parameters pertinent to data reduction.

3.1.1 Functional Description

The major elements of the laser ranging system on-board the Shuttle are shown in Figure 42. The major interfaces are shown to illustrate the more significant functions. The clock subsystem issues a command sequence to the laser transmitter to initiate a pulse at or near an expected time, nominally at a 10 pps rate. The clock also determines both the actual pulse departure time

and the detected return pulse time, and forwards this data to the data assembler and the navigation computer. The navigation computer either accepts or rejects the data point based on several test criteria. If accepted, the measurement is used to update the state vector via a Kalman filter technique. The navigation computer also generates the spacecraft/target range vector (and its derivative) for the next pulse emission time. The magnitude of the range vector is used to set the ranging receiver range gate. The range vector and its time derivative is forwarded to the ARS computer, which rotates both into Shuttle coordinates and converts this data into gimbal position command. The navigation computer outputs range vector data at a 10 sps rate; the ARS computer processes the direction and rate predictions into a gimbal angle trajectory command sequence which is delivered to the pointing control subsystem computer at a nominal 200 sps rate. The pointing control system compares the gimbal positions with the command positions and issues the necessary acceleration commands to the beam steering subsystem torquers. The ARS computer also maintains a current estimate of the optical bench attitude in stellar inertial coordinates.

The laser ranging system is controlled by the command and control system which contains the desired ranging measurement sequence and the necessary navigation data to initialize the navigation computer prior to initial target acquisition.

The RAU is the data and command interface with the Shuttle/Spacelab data and command and control system. Shuttle electrical power is conditioned and delivered to the laser ranging system components by the power conditioning unit.

The thermal control subsystem includes insulation, heaters, coldplates, and thermoelectric heat transfer devices to maintain the components within the desired operating temperature ranges.

Table 4 summarizes a typical laser ranging experiment operation during a ranging pass. The first operation, called pre-align, requires applying power to some of the active components, including the thermal control subsystem, to establish thermal equilibrium in the sensitive elements. The next step is to initialize the navigation computer and commence the navigation function. This includes determination of the desired Shuttle attitude for the ranging pass. The attitude data (stellar inertial coordinates) is forwarded to the Shuttle, which is rotated into the desired attitude for the pass.

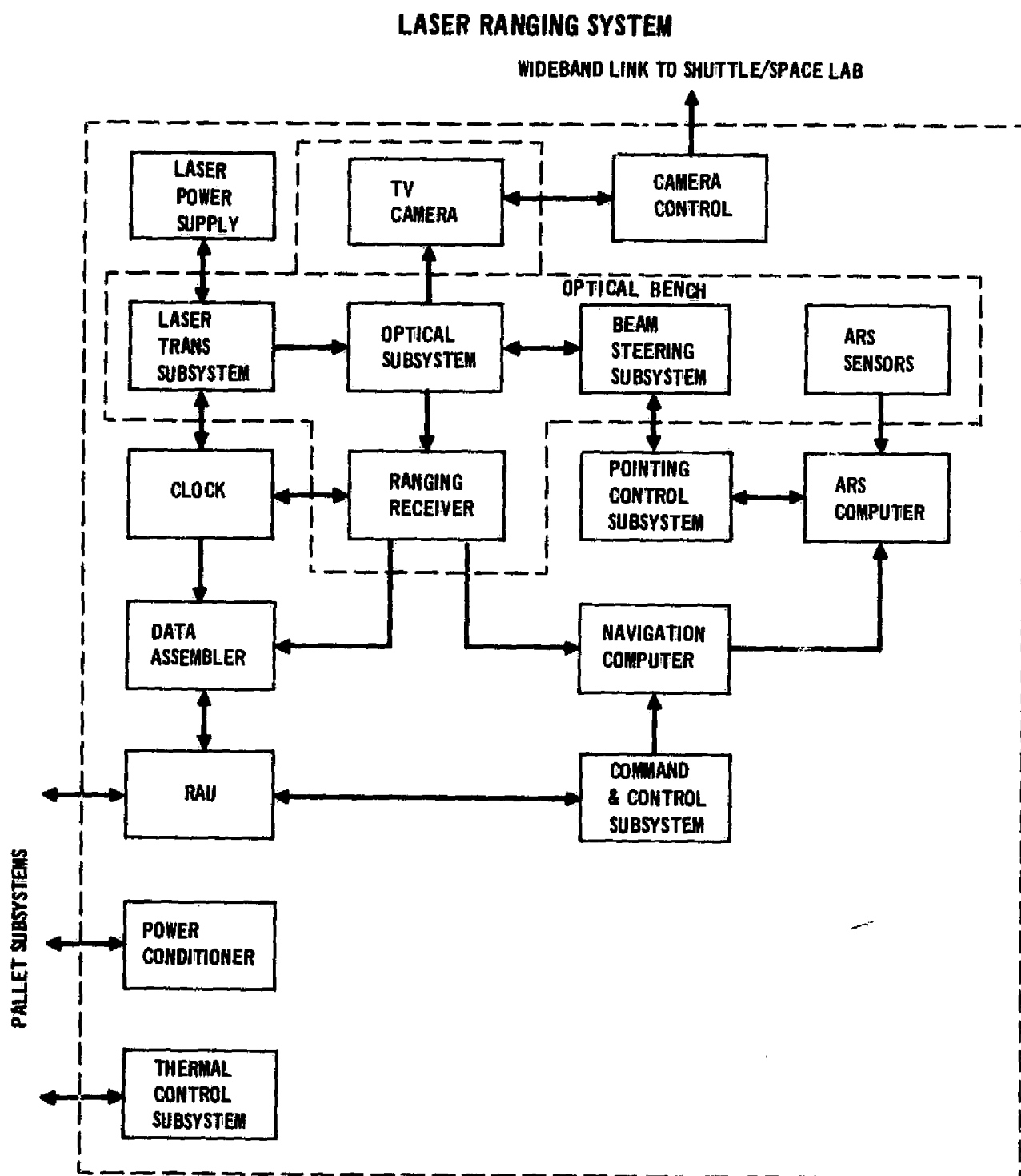


Figure 42

Table 4
TYPICAL LASER RANGING EXPERIMENT

Mode	Activity	Duration	Control
Prealign	Powerup Selected Components	TBD	External Command
Navigation Initialize	Nav. Computer on	TBD	(1) External Cmnd On (2) Transfer Desired Shuttle Orientation
ARS Align	ARS on Star Trackers on	~30 min	Automatic
Coast		~15 min	Automatic
Pre-Acquisition	Gimbals Active Laser On	~20 sec	Automatic
Acquisition	Dither about Nominal Target Position	~10 sec	Preprogrammed Sequence
Verify	Test & Update	~2 sec	Automatic
Ranging	Track/Slew/Track	~260 sec	Preprogrammed Sequence
Post-Pass	Laser Off, Thermal Cooldown	~5 min	Automatic

The next step is to enter the ARS align mode. The star tracker output data is used to determine the actual inertial orientation of the optical bench and to calibrate the ARS gyros drift and scale factors. This mode is maintained until the star trackers line of sight is (effectively) obscured by the earth. At this time the ARS reverts to a gyro only mode until the end of the ranging pass.

The next phase is the preacquisition mode. In this mode, the navigation computer begins the target view computation sequence, and the ARS computer uncages and commands the desired pointing angles for the mirror gimbals. Approximately 20 seconds before the start of the acquisition sequence, the laser transmitter is enabled and thermally stabilized. During this period, the mirror is positioned to the nominal predicted location of the first target to be observed.

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If the target is not detected during the preacquisition mode, the system enters the acquisition mode, which requires dithering the mirror gimbals about the nominal line of sight to the target in a preprogrammed sequence.

Once the target is detected, the system enters the target verify mode. In this mode, the observed detection characteristics are compared to preset criteria, and a valid/invalid decision is made. If a valid target decision is made, the navigation computer is updated and the system enters the ranging mode. If, however, the invalid target decision is made, the acquisition mode is resumed.

An alternate acquisition mode was considered which merged the acquisition and target verify modes. In this alternate concept, a sequential search was performed in space about the nominal line of sight.

However, instead of moving from point to point in the search sequence after a prescribed dwell time, each point would be viewed until either a no target or a target present decision could be made with a prescribed confidence level. This acquisition mode was not considered as suitable for this application as the fixed dwell time concept, since the major acquisition problem is due to atmospheric scintillation, and deep fades can last a considerable time period.

In the ranging mode, a preprogrammed measurement sequence is followed. The transmitter is slewed to illuminate one of the targets in the grid, and ranging measurements are made for N successive pulses, where N is a variable dependent on the number of targets in the grid.

The system is then slewed to the next target in the sequence and the measurement sequence repeated. This sequence of track, slew, track is continued until the last target is no longer in useful view.

The post-pass mode is provided to ensure completion of all buffered functions and to ensure that the laser has had time to cool down. The laser ranging system is subsequently powered down to the hold mode, where only the clock and TCS monitor functions are retained.

3.1.2 Folding Mirror and Pointing Control System

The baseline model for the folding mirror, gimbals, and control system is derived from a similar system under development for an Air Force laser communication program. Figure 43 shows the overall dimensions of the gimbal mirror assembly. It is in a 2 axis roll-elevation configuration with elevation inside

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the roll axis. The gimbal angles are sensed by 20 bit resolution inductosyns, with the pre-amplifiers mounted on the gimbals. The mirror is beryllium for lightness and thermal expansion matching.

GIMBAL MIRROR

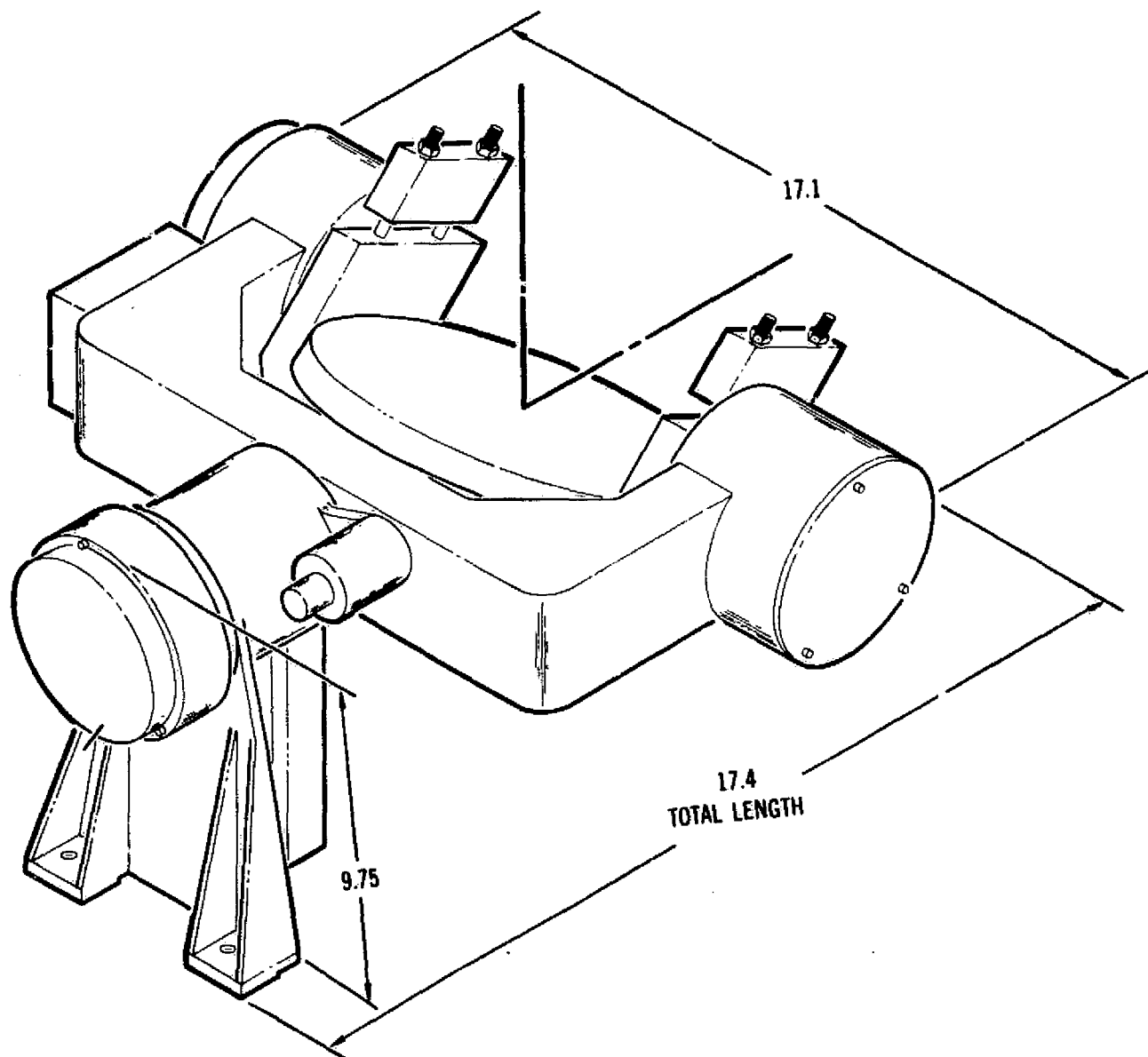


Figure 43

The gimbals are electromagnetically caged (during launch) with power off, and can be uncaged by application of power to redundant D.C. torque motors. Figure 44 shows the assembly details.

The gimbal control system is digitally implemented, excluding the torque motor power amplifiers. Figure 45 shows one axis of the gimbal control system. The normal linear control system is augmented with an acceleration schedule control to facilitate large angular steps. The acceleration schedule control applies maximum acceleration torque in the desired direction until the error signal is less than one half of the initial commanded change in pointing angle. At this time the commanded acceleration torque is reversed. The acceleration schedule is terminated when either the error signal changes sign or a computed time has elapsed. The (conceptual) switches are then positioned to enable the normal linear mode control function.

The control system from which this model is drawn has been demonstrated in both simulation and brass-board testing to attain ~ 2 μ rad rms open loop pointing stability^(*), but the design application required only very slow slewing rates. The major change to the reference model, therefore, is to adapt to our requirement for minimum time to slew from one target in a grid to another. A number of factors must be considered in order to establish the feasibility of these changes and the expected time to settle after a sizable angular step.

The first concern is the ability of the inductosyns to follow relatively large angular rates. The inductosyn is an incremental counting device which maintains the 20 bit accuracy by counting changes (+) in the least significant bit. The subsystem presently used has an error free maximum angular rate of ~ 0.06 rad/s ($3.4^\circ/\text{s}$). If the angular rate exceeds the error free rate, the output lags the shaft angle position. The maximum lag is ~ 700 μ rad. If the angular rate exceeds about 7 rad/s ($400^\circ/\text{s}$), a loss of reference occurs and the gimbals must be stopped to reestablish the actual position.

The maximum angular acceleration for the existing system gimbals is ~ 8 rad/s² in elevation and 2 rad/s² in roll. The maximum angular step for the elevation gimbal is on the order of 0.7 rad, thus the peak angular velocity would be on the order of 2.4 rad/s, well below the angular velocity at which loss of reference would occur. The maximum angular step in the roll axis is less than

* MDC Report E1492, Page 127

GIMBAL MIRROR

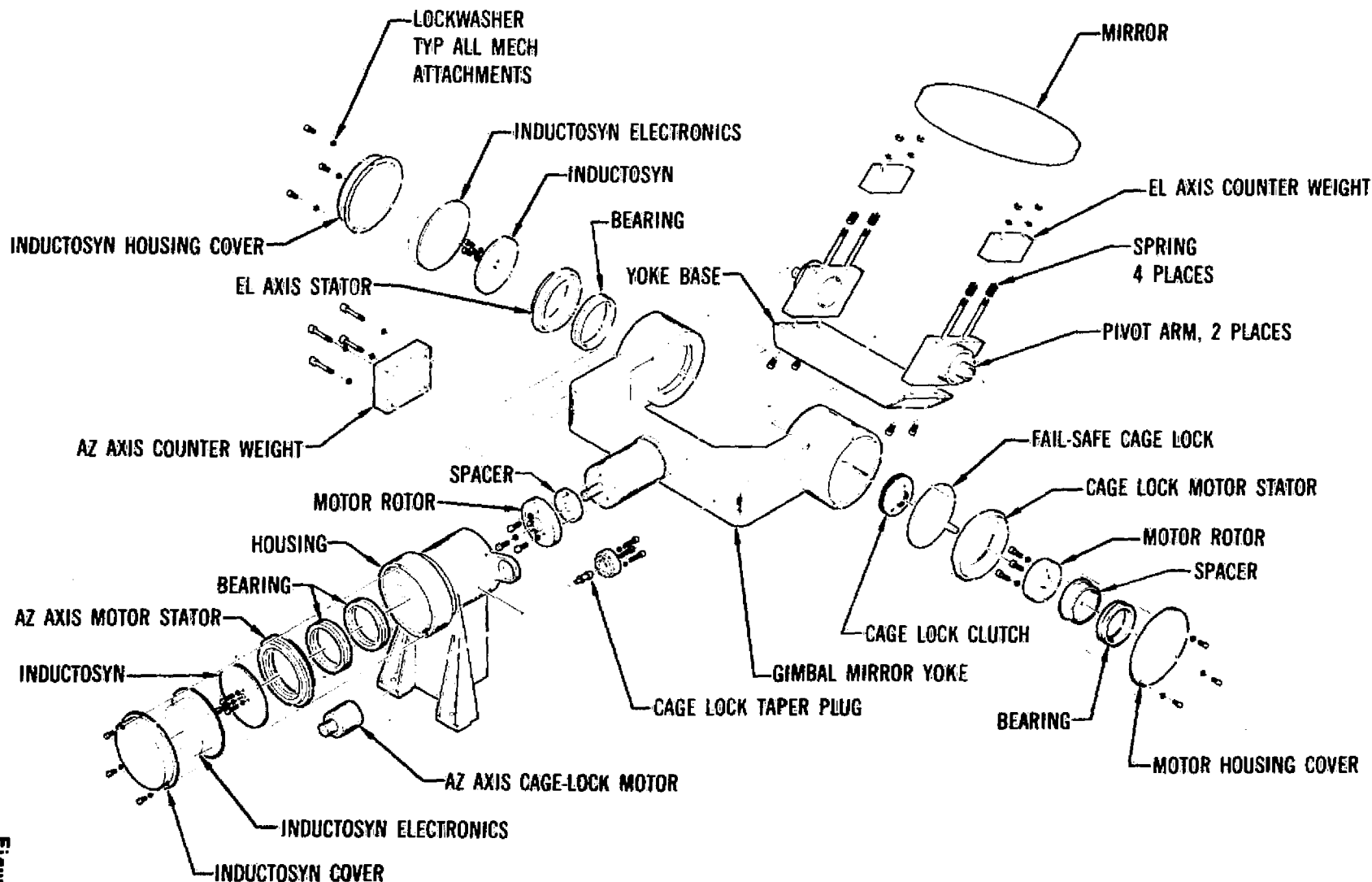


Figure 44

STAND ALONE GIMBAL CONTROL SYSTEM

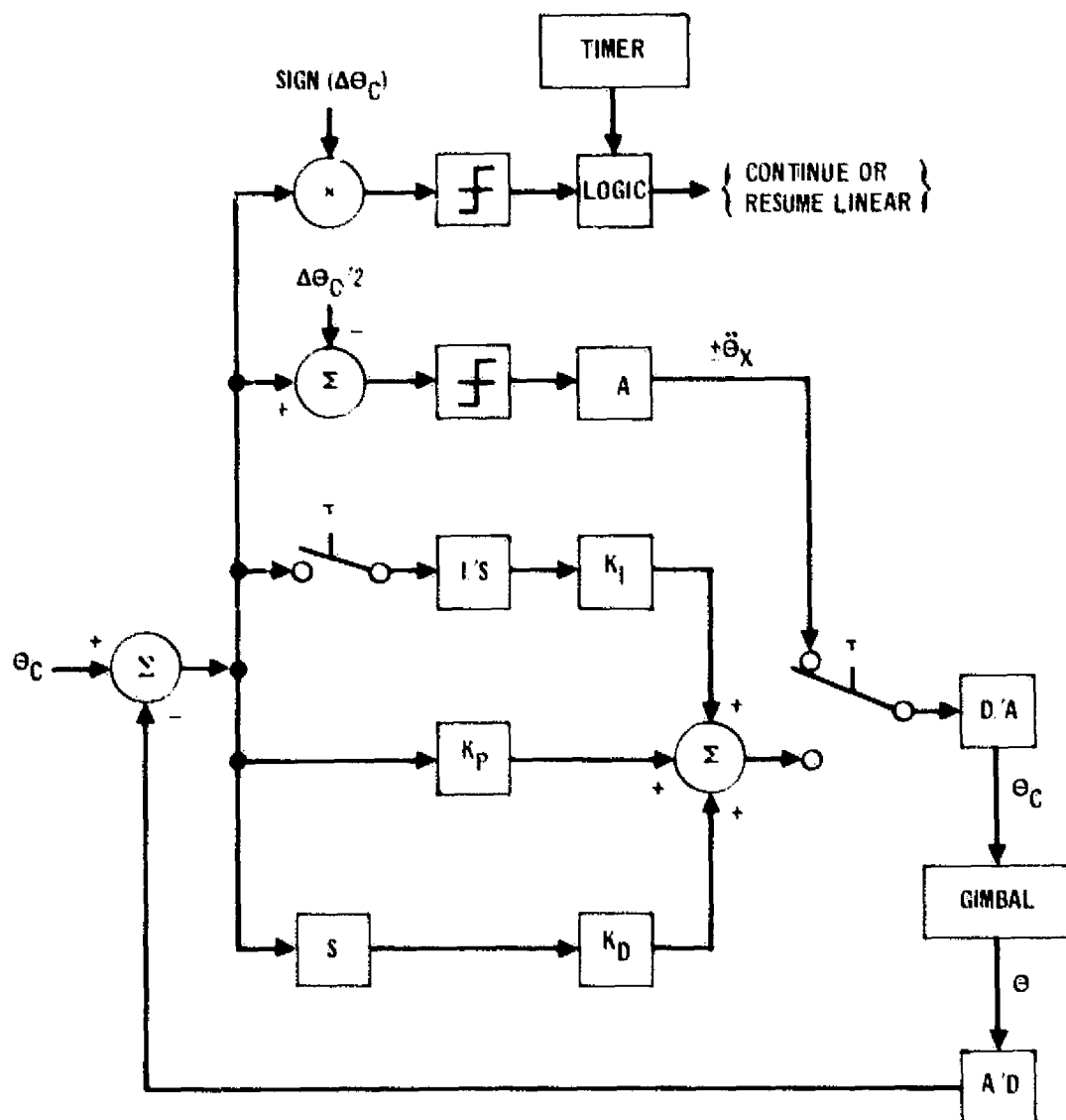


Figure 45

π radius, thus the maximum angular velocity is less than 25. rad/s, also well below the maximum.

Optical shaft encoders are an alternate sensing system which have been considered for this application. A candidate 20 bit optical encoder senses 13 bits directly and the 7 least significant bits using fringe techniques. The maximum error free angular velocity ($\pm 1/2$ LSB) is $6^\circ/s$ (.1 rad/s). (Unlike the

inductosyn, the angular position is a direct readout, thus when the angular velocity decreases below the $6^\circ/\text{s}$ rate, full angular position accuracy is immediately restored). At angular rates above $6^\circ/\text{s}$, the optical shaft encoder output becomes increasingly inaccurate, with the error doubling for each factor of two in angular rate.

Thus, we conclude that the most significant problem with either type of encoder is that our ability to determine when the gimbal has reached the half-way point is decreased, compared to static conditions, at large angular velocities.

The next question to be addressed is the control technique to be employed. The reference system uses an all digital control system implemented with a microcomputer. Consequently, a change in control law simply requires programming a new memory chip.

The reference control system employs a 5 ms cycle time. The cycle time can be reduced, if necessary, at the expense of increased power consumption. At a maximum angular velocity of $\sim 2.4 \text{ rad/s}$, the position of the gimbal would change $\sim 12 \text{ mrad}$ between successive samples, thus we see that the loss of accuracy due to output lag for the inductosyn (or resolution for the optical encoder) is trivial in comparison.

As presently planned, the acceleration schedule changes sign when the error signal is less than one-half of the commanded angle change. Thus, there is a lag between the actual half-angle crossing and the commencement of the negative acceleration schedule of up to one cycle time.

This lag results in a residual angular velocity at the end of the acceleration schedule, virtually insuring an overshoot condition. The residual angular velocity is a function of both the commanded step size and the cycle time. This residual angular velocity increases the time required for the gimbal to settle to within acceptable pointing accuracy (on the order of $50 \text{ } \mu\text{rad}$). The settling time can be quite long if the residual velocity causes the gimbal to significantly overshoot the linear control range.

The next concern is to examine the situation where the target gimbal angle is not stationary. Consider the situation where the Shuttle has a non-zero angular rate in the gimbal axis coordinates. Thus, the commanded gimbal

angle is a time varying angle defined as $\theta_c = \theta_0 + \dot{\theta}_c t$. An ideal acceleration schedule would accelerate to time t_1 , and then decelerate till time t_2 , the gimbal angle and angle rate would match the target angle and angle rate if,

$$t_1 = -\frac{\theta_c}{\alpha} + \frac{\dot{\theta}_c}{\alpha} + 1/2 \left(\frac{\dot{\theta}_c}{\alpha} \right)^2$$

and

$$t_2 = -\frac{\theta_c}{\alpha} + 2 \left(\frac{\dot{\theta}_c}{\alpha} \right) + 1/2 \left(\frac{\dot{\theta}_c}{\alpha} \right)^2$$

where the sign of α is chosen to make t_1 (hence t_2) positive and real.

When the angular step size is large and the target angular rate modest, we find that the error signal ($\epsilon = \theta_c - \theta$) at time t_1 is,

$$E = \theta_0/2 + \dot{\theta}_c^2/4\alpha$$

Consequently, we see that the half-angle step strategy results in a small switching lag (compared to an ideal acceleration schedule).

There is another factor, however, which adds to the utility of the half angle concept. The torque actually applied to the gimbals for a fixed drive voltage varies somewhat with gimbal angle. This torque ripple results in a potentially significant dispersion of angular position and velocity at the end of a time ordered acceleration profile. The half angle switching strategy tends to minimize this dispersion. However, it is this dispersion which requires adding the timer to terminate the acceleration schedule, since a greater than expected torque during the latter half of the schedule could result in cancellation of the lag effects to the extent that no overshoot occurs.

A simplified simulation was constructed to test the performance of the gimbal control system. Torque ripple was modelled as a random process; encoder error was also modeled as a random process with an angular velocity dependent variance. Figure 46 shows the response of the roll gimbal to a 0.15 rad step command with zero and ± 50 mrad/s command rates. The gimbal settled to within 50 μ rad within approximately 0.7 s. The control loop was operated with a 5 ms cycle time. Figure 47 shows similar curves for the inner gimbal, also at a 5 ms cycle time. In this figure, more pronounced overshoot can be observed; even though the first crossing occurred much sooner than in the roll gimbal case, the overall time to settle was only slightly less

RESPONSE OF ROLL GIMBAL TO A 0.15 rad STEP COMMAND

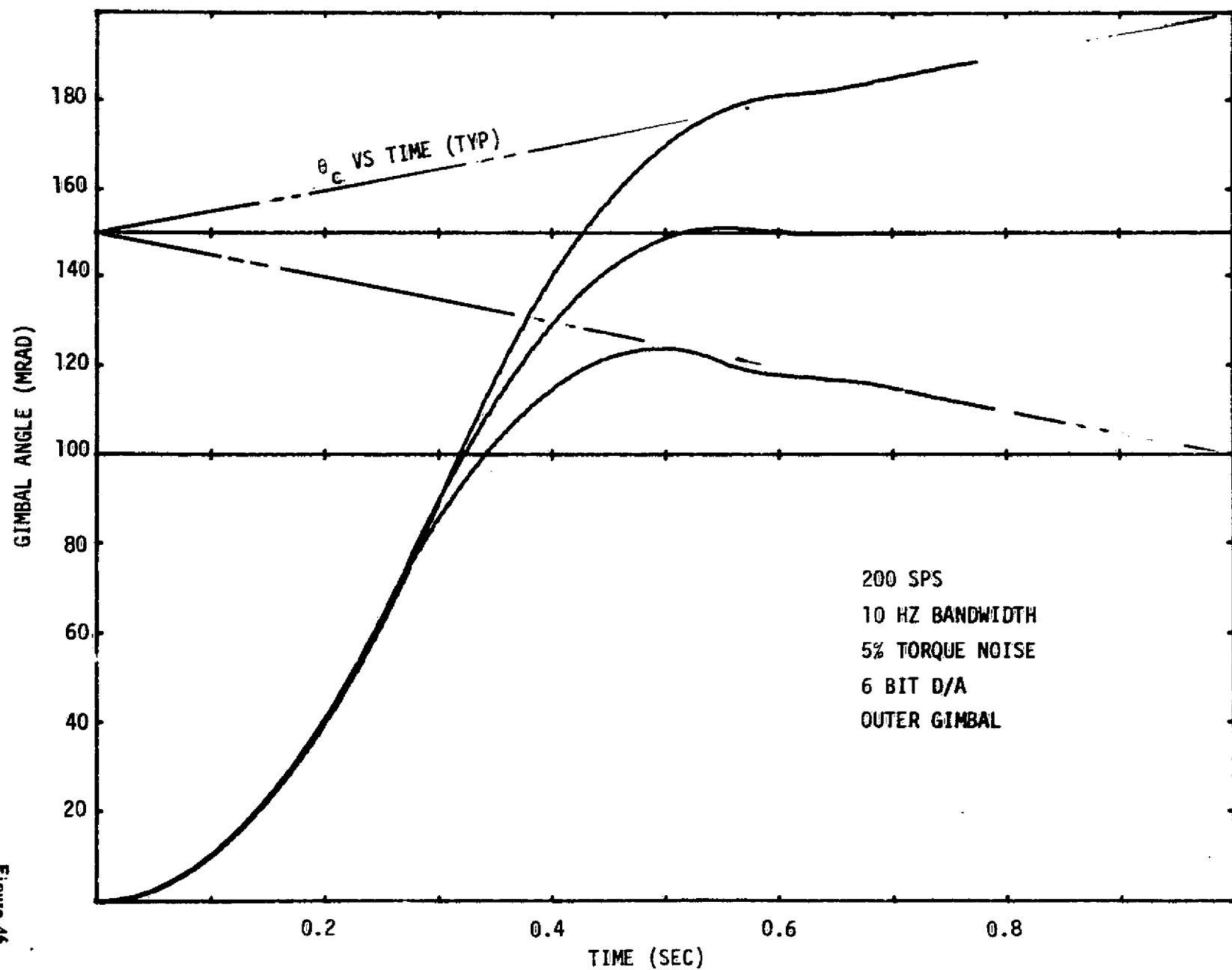


Figure 46

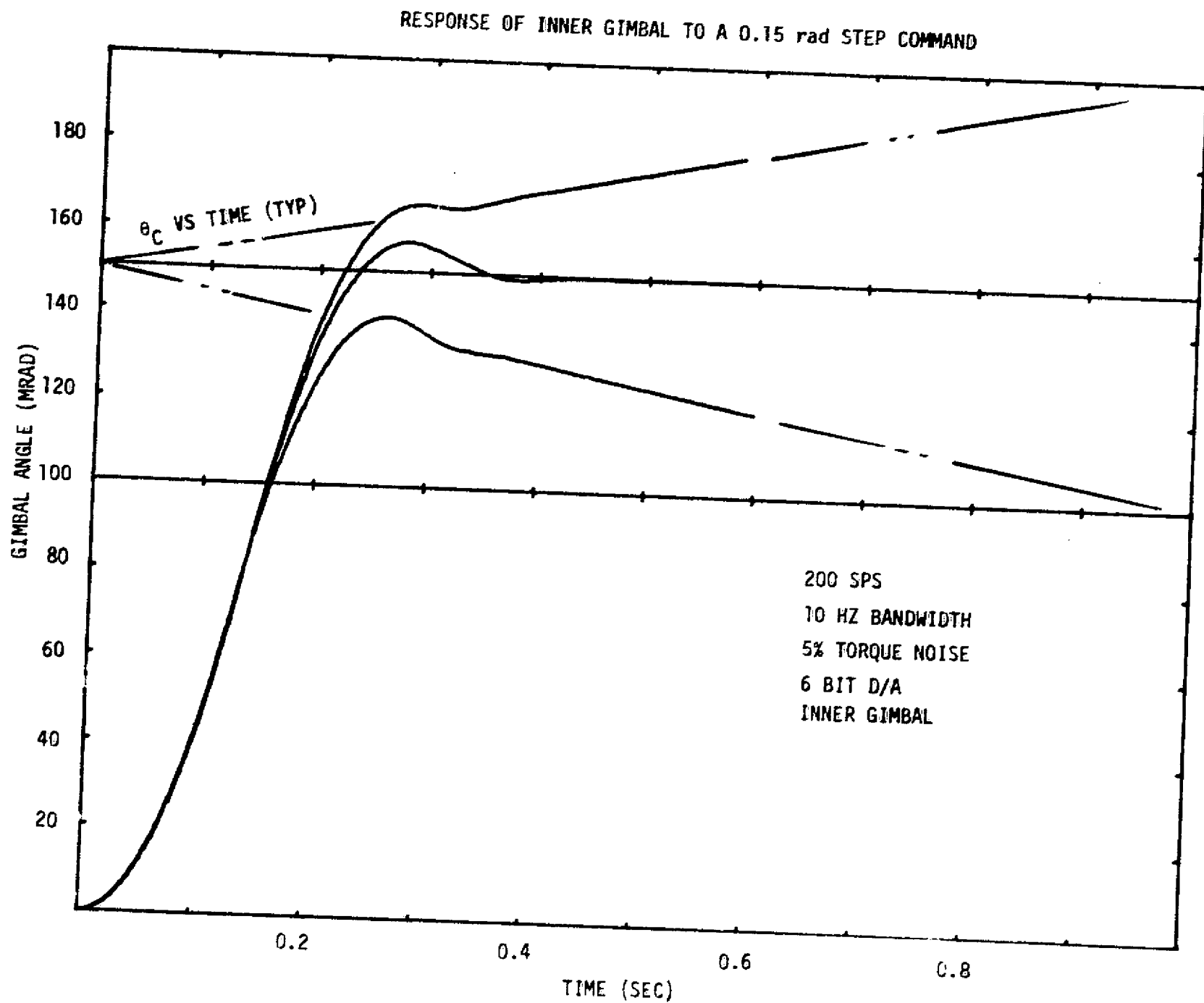


Figure 47

($\sim .55$ s). Figure 48 shows the response of the inner gimbal to a 150 mrad step with + or - 50 mrad/s rate, but with a 1 ms cycle time. The large overshoot observed in the preceding figure has been eliminated, and the gimbal settled to within 50 mrad in ~ 0.35 s.

In this simulation, the gimbals were modeled as frictionless and linear. This assumption does not materially affect the dynamic performance except in the small angular rate regime where breakout forces are encountered.

In conclusion, a modified version of a pointing control system currently under development has been evaluated for the laser ranging experiment, and found to be reasonably close to meeting all pointing requirements. Additional modifications to increase the available torque and decrease the time to slew and settle are certainly plausible; considerable additional evaluation would be required, however, to verify the fine pointing capability if larger torques are used.

3.1.3 Opto-Mechanical Subsystem

The opto-mechanical subsystem is shown schematically in Figure 49. The laser output is passed through a frequency doubler and then expanded to approximately 2 cm diameter and defocussed slightly to obtain 0.5 mrad divergence. The doubled output is reflected by a dichroic mirror, and directed to an alignment mirror, which directs the beam to the final folding mirror. The final folding mirror folds the transmit beam to be aligned with the primary telescope axis. A small portion of the transmit beam is extracted at the final folding mirror and used to determine the overall transmit beam alignment.

Another small fraction of the transmit energy is diverted through a neutral density filter and reflected to the high speed photomultiplier tube in order to obtain an estimate of pulse departure time.

The receive optics path includes a bifurcating mirror which allows a star sensor to be used to calibrate the gimballed folding mirror (not shown). The central part of the receiver field of view passes through the bifurcating mirror, encounters two torque motor beam steering elements, and a narrow band optical filter. Provisions were made to insert either a shutter or a variable density filter. The receive path includes another bifurcating mirror, which splits off

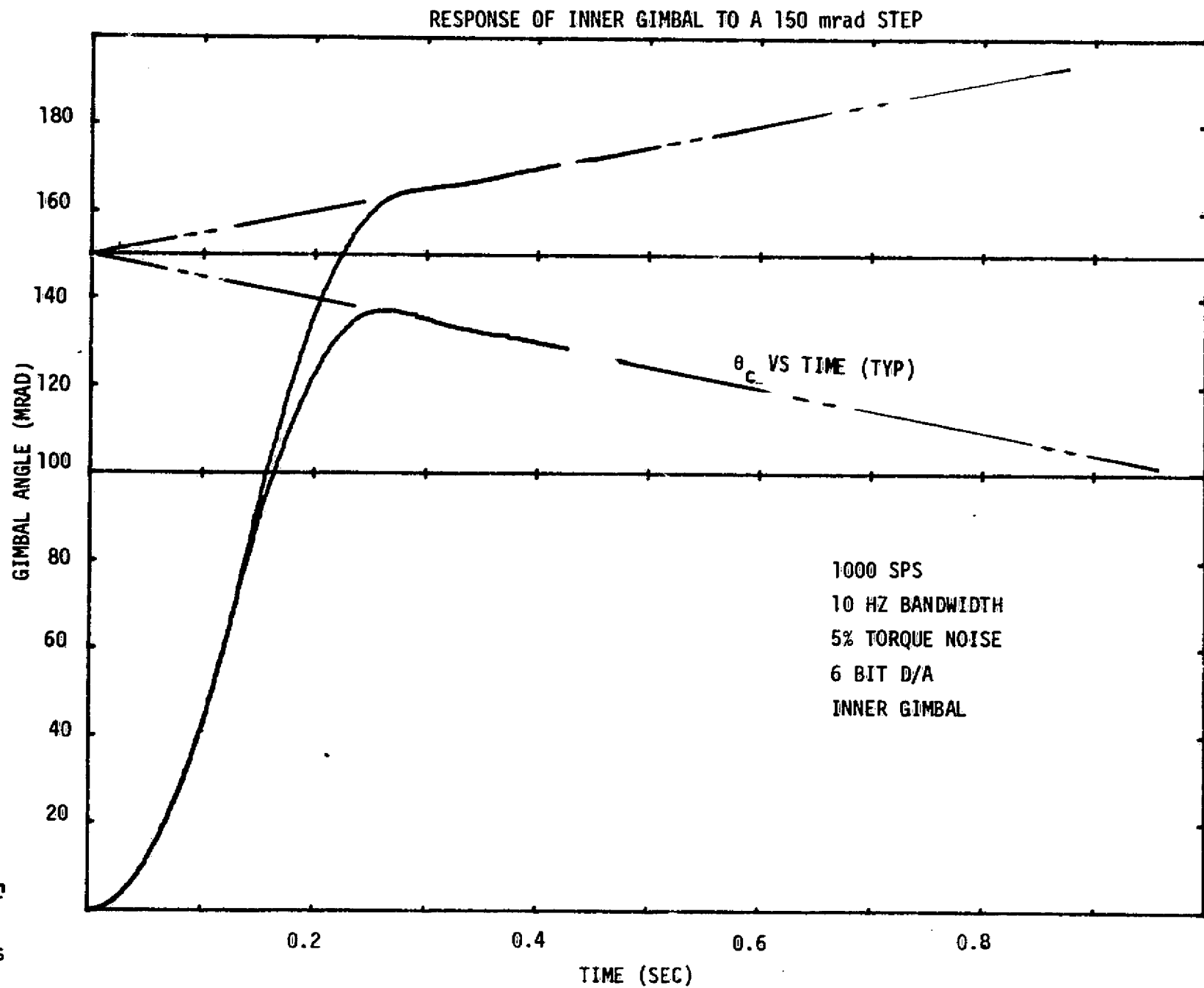


Figure 48

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OMS SCHEMATIC

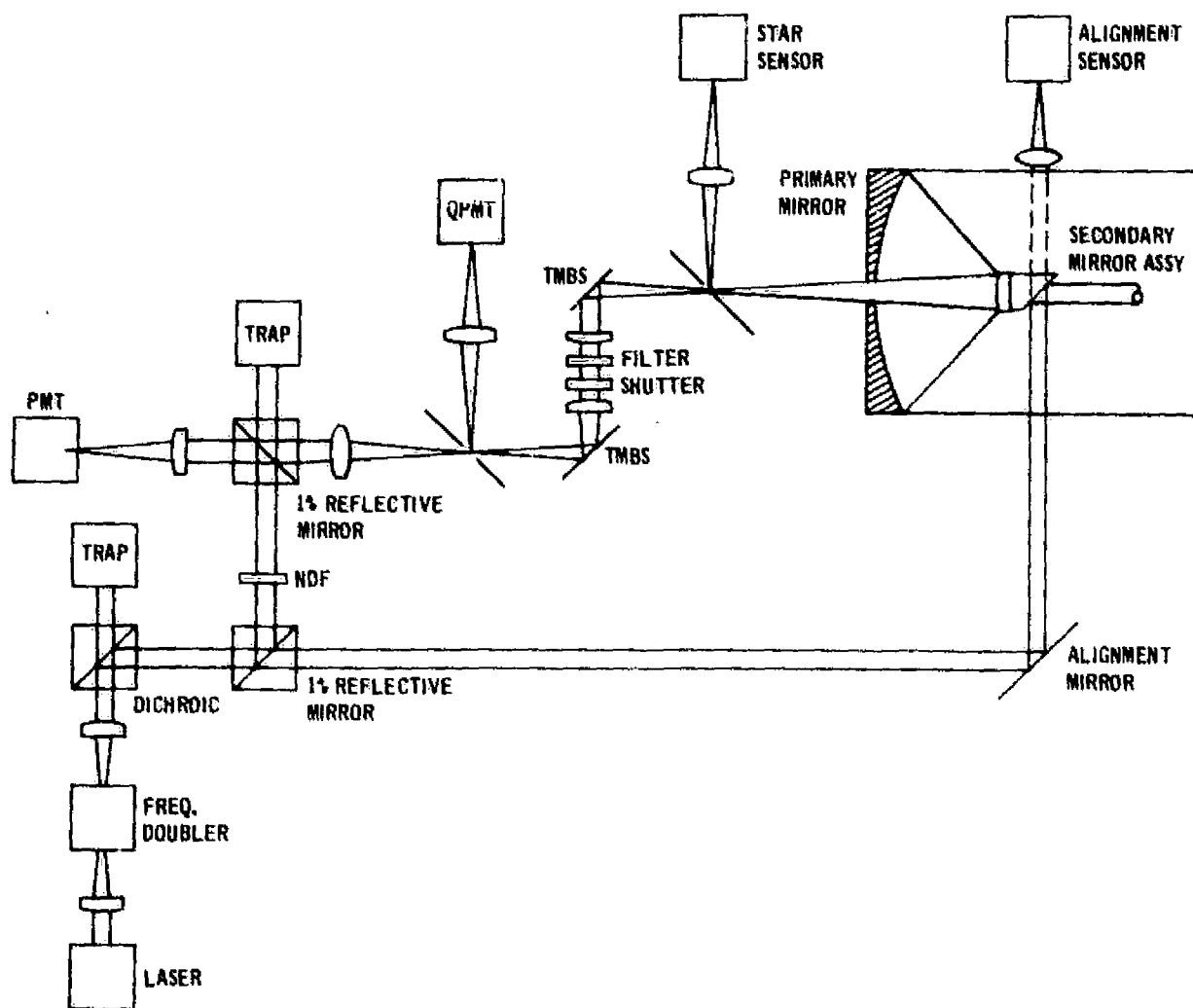


Figure 49

the outer edges of the inner field of view to a quadrant PMT for gross receiver boresight error compensation. The central 0.5 mrad field of view is then focused on the ranging detector.

The collecting and resolving power of the telescope is such that the star sensor path could be split into two paths (50% reflective mirror), with the second path directed to a television camera to enhance the experiment monitoring capability of the crew.

Various concepts for on-orbit calibration were explored to determine relative accuracy and feasibility. The first phase of the on-orbit calibration process would be to calibrate the gimbaled folding mirror and receiver primary focal

plane using the focal plane star sensor. The attitude reference system must be operating during this period to relate star sensor output to the navigation base attitude. The boresight location of the inner field of view can be determined by driving the gimbals such that a star traverses through the inner field of view in a prescribed manner, thus the star sensor drop out during the traverse is indicative of the centering of the inner field of view and the star sensor field of view.

If the narrow band optical filter is removable, and the PPMT is replaced with a CCD type star sensor, a further fine resolution calibration of the torque motor beam steering units can be performed, resulting in a very precise centering of the inner field of view with the outer field of view and the gimbals.

The major problem remaining is to verify the alignment of the transmit beam with the calibrated receiver boresight. The most attractive approach seems to be to locate a collimator on the optics bench, and to command the gimbal system to a preselected position such that the transmit beam illuminates the collimator. Of course, this would be beyond the region where the gimbal calibration can be verified by stellar observation, but the probable error resulting from extrapolation is likely to be much smaller than the alignment tolerance. Once this calibration is completed, the alignment sensor is also calibrated, and subsequent short term alignment variations can be removed using the alignment mirror adjustment capability.

The receiver and gimbal calibration can be readily accomplished during the gyro drift and scale factor calibration period preceding a ranging pass. The transmitter calibration can be performed in the short time interval between initiation of laser operation and the start of the ranging pass. The transmitter is turned on and allowed to stabilize, and then the alignment verified (or corrected). The gimbals are then commanded to the nominal acquisition attitude and normal operation begun. The location of the collimator is chosen to minimize the gimbal slew required to move to the acquisition orientation.

3.1.4 Interfaces

The experiment interfaces are with the pallet and pallet subsystems, and ultimately with the Spacelab and Shuttle subsystems. The optical bench is supported on a sill level platform to obtain an adequate field of view for

the ranging experiment. The balance of the laser ranging system components are mounted on coldplates attached to the pallet structure, as shown in Figure 40.

Most of the LRS components can be mounted directly on coldplates serviced by the pallet freon loop. A few components, such as the laser transmitter, will require more precise thermal control than provided by the standard pallet thermal control system. For these components, a thermoelectric unit is inserted between the coldplate and the unit. Heaters and insulation are provided to maintain the optical bench at a stable temperature.

The LRS data interface with Shuttle/Spacelab is through a standard RAU. As presently planned, data will be transferred to the Shuttle/Spacelab in several manners. Ranging data will be transferred in discrete packages, assembled by the LRS. Each package will be on the order of two hundred bits, and will be output at a ten packages per second rate. Performance analysis data will also be forwarded in discrete packages, on the order of 5000 bits per package, also at the ten package per second rate. Engineering data will be assembled and forwarded at a nominal 400 bps rate during the operating period. The data load is summarized in Table 5. Ranging data is the data necessary to reduce the range measurements on the ground after the pass. Performance analysis data is intended to give as complete a picture of the actual experiment performance as possible, and may be monitored in part by the experiment operator. Engineering data is intended to monitor the performance of the support subsystems, and is expected to be monitored by the experiment operator, as well as being forwarded to the ground.

Table 6 summarizes the expected data load from a single LRS ranging pass.

Table 7 summarizes the data load if constrained to Shuttle standard data rates.

It may not be necessary to dump the performance analysis data at the LRS pulse rate, which could reduce the peak data rate and total data load considerably.

Another interface with the Shuttle/Spacelab is the LRS input data and command interface. This data is summarized in Table 8.

Electrical power received from the Shuttle/Spacelab is conditioned and delivered to the various components. The laser power supply is the single largest load, about 390 watts prime power is the estimated requirement. The thermoelectric coolers will require only modest amounts of power, except under extreme opera-

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TABLE 5
DATA LOAD SUMMARY

I RANGING DATA SUMMARY

FUNCTION	(NO.)	(BITS)	BIT RATE	
			ACQ	PASS
TARGET LABEL	1	8	80	80
ACQ. GRID LABEL	1	8	80	80
T _x	1	54	540	540
T _R	1	54	0	540
SIGNAL STRENGTH	1	12	0	120
GATE WIDTH	1	16	160	160
GIMBAL READOUTS	2	19	380	380
SUBTOTAL			1240	1900
OVERHEAD			138	211
TOTAL			1378	2111

II PERFORMANCE ANALYSIS DATA

FUNCTION	(NO.)	(BITS)	BIT RATE	
			ACQ	PASS
STATE VECTOR	13	32	4160	4160
COVARIANCE MATRIX	91	32	0	29120
RANGE ESTIMATE	1	32	320	320
RANGE RESIDUAL	1	16	0	160
DRAG ESTIMATE	1	16	160	160
ATM CORRECTION	2	16	320	320
TARGET POINTING VECTOR	3	32	960	960
TARGET POINTING RATE	3	16	480	480
QUAD. DETECTORS	4	8	320	320
BASEPLATE ORIENTATION	3	32	960	960
GIMBAL ACCEL. CMNDS	2	8	3200	3200
SUBTOTAL			10880	40160
OVERHEAD			1209	4462
TOTAL			12089	44622

ENGINEERING DATA

FUNCTION	(NO.)	(BITS)	(SAMPLE RATE)	BIT RATE
GMT	1	24	1/60	0.4
TEMPERATURE	100	8	0.1	80
VOLTAGE	100	8	0.1	80
PRESSURE	10	8	0.1	8
DISCRETES	20	1	1.0	20
EVENT TIMES	1	32	0.1	3.2
ARS DATA (EXP.)	3	24	0.1	7.2
ARS DATA (SHUTTLE)	3	24	0.1	7.2
INS DATA (SHUTTLE)	6	20	0.1	12.0
THRUSTER FIRING LABEL	1	16	0.1	1.6
THRUSTER FIRING TIME	1	24	0.1	2.4
THRUSTER FIRING DURATION	1	12	0.1	1.2
SPARE CHANNELS				136.8
SUBTOTALS				360
OVERHEAD				40
TOTAL				400

TABLE 6
DATA LOAD SUMMARY

<u>MODE</u>	<u>DURATION</u>	<u>I BPS</u>	<u>II BPS</u>	<u>III BPS</u>	<u>TOTAL BPS</u>	<u>TOTAL BITS MBITS</u>
PRE-PASS	45 MIN.	0	0	400	400	1.08
ACQ.	60 SEC	1378	12089	400	13467	0.808
RANGING	260 SEC.	2111	44622	400	46733	12.15
POST-PASS	5 MIN.	0	0	400	400	0.12
TOTAL DATA LOAD/PASS						14.16

TABLE 7

DATA LOAD-STANDARD SHUTTLE BIT RATES

<u>MODE</u>	<u>DURATION</u>	<u>ACTUAL DATA RATE BPS</u>	<u>STANDARD BIT RATE BPS</u>	<u>TOTAL DATA LOAD MBPS</u>
PRE-PASS	45 MIN.	400	400	1.08
ACQ.	60 SEC.	13467	25600	1.536
RANGING	260 SEC	46733	51200	13.312
POST-PASS	5 MIN.	400	400	0.12
TOTAL DATA LOAD/PASS				16.048

TABLE 8
COMMAND LOAD SUMMARY

GROUND TO LRS

<u>ITEM</u>	<u>BITS/WORD</u>	<u>NO. OF WORDS</u>	<u>NO. BITS</u>
WAKE UP TIME	24	1	24
TARGET LABEL	8	9	72
TARGET LAT.	32	9	288
TARGET LONG.	32	9	288
TARGET COV. MATRIX	32	45	1440
ACQ. SEQUENCE	8	200	<u>1600</u>
TOTAL			3712

SHUTTLE/SPACELAB TO LRS - INITIALIZE

ON/OFF/MODE	8	10	80
EPHEMERIS	32	6	192
EPOCH	24	1	24
ARS	24	3	72
GMT	24	1	<u>24</u>
			392

LRS TO SHUTTLE/SPACELAB.

ATTITUDE COMMAND	24	3	72
MODE/DATA RATE	8	1	8

ting conditions (max. coolant temperature). Assuming the laser is to be maintained at a 20°C heat exchanger input condition, the peak 40°C coolant temperature will require pumping ~300W up 20°C. At that temperature differential, a single stage cooler pumps at about a 1.75 ratio, thus the cooler would require about 170 watts to pump 300 watts across a 20°C differential. At the freon system lower temperature level, the cooler drive can be reversed to maintain

a steady 20°C heat exchanger input. The overall experiment power consumption at full operating condition is estimated to be 854 watts. The estimated energy consumption profile is shown in Table 9. About 6.7 kw-hrs are consumed by the frequency standard and clock if operated continuously. The energy consumed per pass is about 0.42 kw-hrs, most during the 45 minute alignment period preceding the actual ranging pass. For planning purposes, it was assumed that ten ranging passes, total, would be exercised to cover the ground truth sites and to calibrate the system, resulting in a net energy consumption of 10.9 kw-hours. Table 10 summarizes the estimated weight and power consumption and dissipation of the various subsystems and components. The ARS sensors and the laser power supply are expected to interface directly with the Shuttle/Spacelab power bus. All other components are powered by the power conditioner, which converts Shuttle/Spacelab bus power into regulated power at the appropriate voltages.

TABLE 9
POWER PROFILE ESTIMATE

PRIME POWER INPUTS	POWER VS MODE (DURATION)				ENERGY CONSUMED 10 PASSES (KW-HR)
	OFF (7 DAYS)	ALIGN (45 MIN)	PASS (280 SEC)	POST-PASS (5 MIN)	
LASER PWR SUPPLY	0	0	390W	0	.3
ARS SENSORS	0	66W	66W	66	0.6
POWER CONDITIONER	40W	359W	398W	359W	10.0
TOTAL	40W	425W	854W	425W	10.9
ENERGY - 10 PASSES	6.7	3.2	0.7	0.3	10.9

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TABLE 10
EQUIPMENT CHARACTERISTICS SUMMARY

<u>ITEM</u>	<u>WEIGHT ESTIMATE (LBS)</u>	<u>DISSIPATED POWER (WATTS)</u>	<u>COOLING POWER (WATTS)</u>	<u>PRIME POWER (WATTS)</u>
ARS SENSORS	34	66	0	66
LASER TRANSMITTER	20	300	0-170	0
LASER POWER SUPPLY	10	90	0	390
FREQ. STANDARD	20	20	0-11	0
CLOCK	10	10	0	0
OPTICAL RECEIVER	30	10	0	0
POINTING CONTROLLER	25	20	0	0
GIMBAL MIRROR ASSY	35	0	0	0
OPTICAL BENCH	18	0	0	0
NAV. COMPUTER	7	15	0	0
ARS COMPUTER	7	15	0	0
C&C UNIT	5	5	0	0
DATA ASSEMBLER	5	10	0	0
SUBTOTALS	226	561	0-181	456
TCS	15	20	0	0
POWER CONDITIONER	25	92	0	398
TOTAL	266	673	181	854

3.2 SIPS Mounted Experiment

The configuration of the SIPS mounted experiment is illustrated in Figure 50. The experiment hardware is shown in two basic groupings, those components mounted on the optical bench, and those components attached to the SIPS cannister directly. The optical bench is centered in the cannister, and mounted to the gimbal frame with a three point suspension. The optical bench is separated from the balance of the LRS components by a thermal curtain to minimize radiative heat transfer.

A typical ranging pass follows the following sequence. The SIPS is deployed and oriented for a prepass cool-down period of currently unspecified duration. After thermal stabilization has been achieved, the system is ready for the next ranging pass, and a wait mode is initiated. Approximately 45 minutes prior to the actual ranging pass, the LRS is selectively powered up and the SIPS is commanded to point to several stars to reduce the initial attitude reference uncertainty to acceptable limits. A programmed sequence of stellar sightings is used to calibrate the ARS gyros. The Shuttle/Spacelab is then commanded to the desired inertial attitude for the ranging pass, and the SIPS is oriented to the nominal inertial position for acquisition.

Thirty seconds before the first target comes into view, the laser is energized and allowed to stabilize. When the first target is expected to be in view, an acquisition sequence is initiated. A short target verify period follows initial target detection. The experiment then enters the normal ranging mode, which continues until the last target is no longer in view. At this time, the LRS is shut down, and the SIPS returns to the cool-down mode.

Data accumulated during the pass is recorded for subsequent transmission to the ground. A typical individual measurement record includes both the time the pulse left and the time of arrival of the reflected pulse, gimbal angle data, a target label, and miscellaneous system parameters pertinent to data reduction.

3.2.1 Functional Description

The major elements of the laser ranging system are shown in Figure 51. The major interfaces are shown to illustrate the more significant functions and data flow requirements. The clock subsystem issues a command sequence to the

LASER RANGING EXPERIMENT SIPS ACCOMMODATION LAYOUT

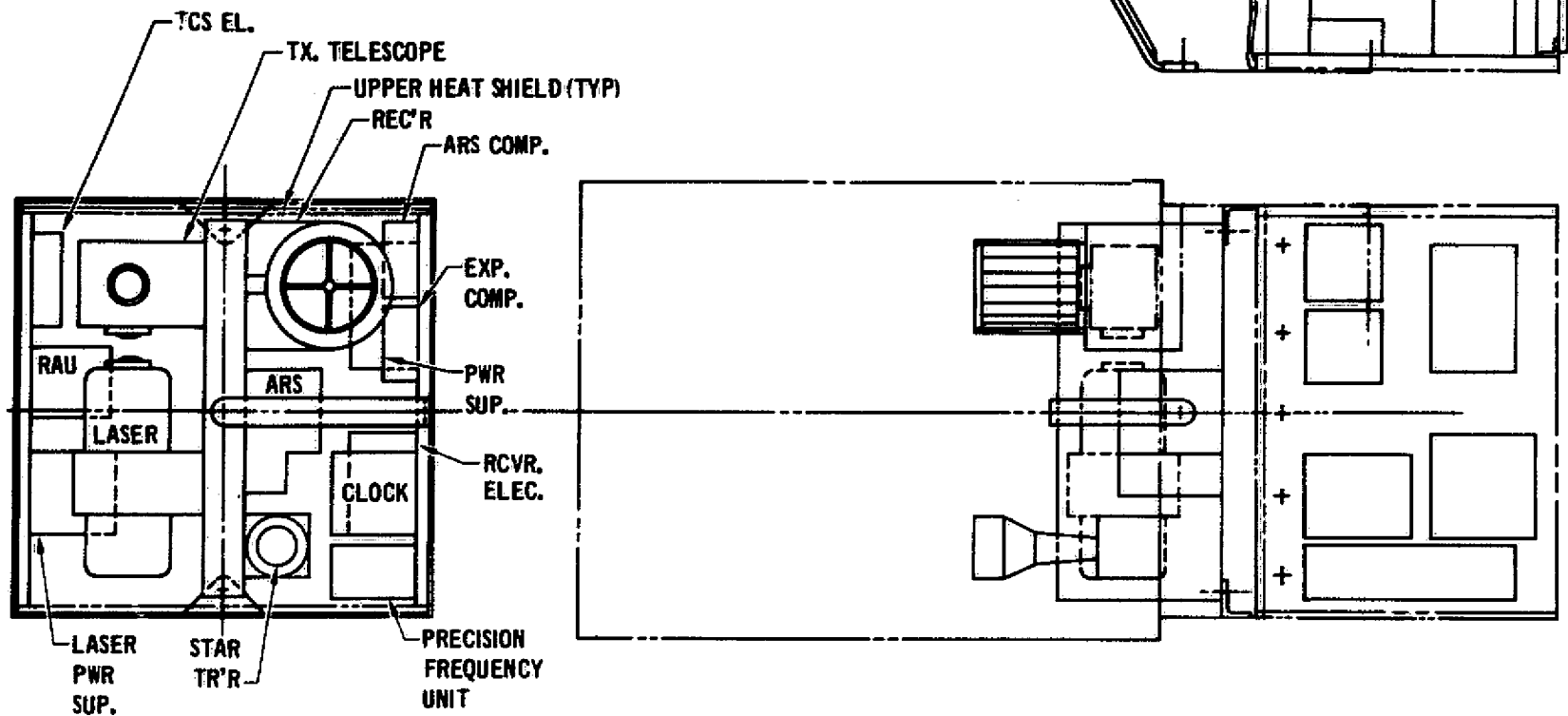


Figure 50

LRS CANNISTER MOUNTED EQUIPMENT

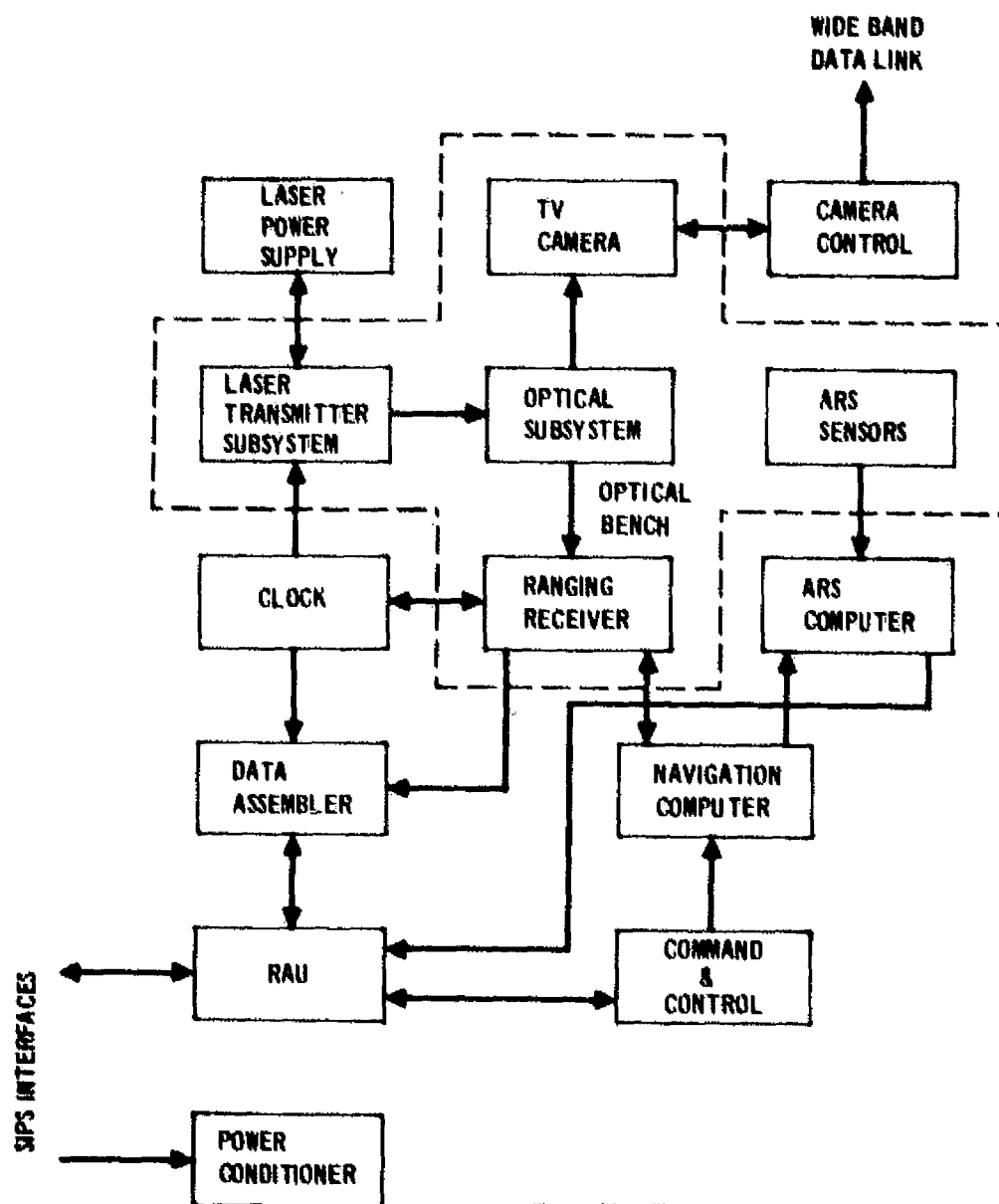


Figure 51

laser transmitter subsystem to initiate a pulse at, or near, an expected time, nominally at a 10 pps rate. The clock subsystem also determines the actual pulse departure time and the time of arrival of the reflected pulse. These data are forwarded to the data assembler and the navigation computer. The navigation computer either accepts or rejects the data point based on several test criteria. If accepted, the measurement is used to update the state vector via a Kalman filter technique.

The navigation computer also generates a target range vector and range rate vector for the next pulse emission time and forwards this data to the ARS computer. The magnitude of the range vector is used to set the range gate in the ranging receiver. The range and range rate vectors are processed in the ARS computer to determine the current pointing error and the command sequence necessary to drive the SIPS gimbals to minimize the pointing error at the next pulse emission time. This control data must be output at a relatively high rate to ensure smooth response of the SIPS gimbal control system. Note that the actual gimbal angles are needed by the navigation computer (and the data assembler) in order to account for the range bias due to offset from the Shuttle center of gravity.

Also shown is a television camera and control to enhance the ability of the experiment controller to monitor and control the experiment operation.

The laser ranging experiment is conducted during a number of discrete ranging opportunities, which occur when the ground track of the Shuttle passes sufficiently close to a target grid. A typical ranging pass begins about 45 minutes prior to the point of closest approach. The first step is to power up the clock, the attitude reference system, and the navigation computer. The clock is initialized to the Shuttle GMT. The ARS is initialized to the Shuttle reference, rotated through the SIPS gimbal angles. The SIPS is then commanded to a preprogrammed series of stellar sightings to refine the ARS attitude reference and to calibrate the gyro drift and scale factor. At a suitable time prior to commencement of the ranging pass, the navigation computer determines the desired Shuttle attitude, in stellar inertial coordinates, and commands the Shuttle to maneuver to and maintain this attitude. The SIPS is then commanded to the acquisition orientation, and the normal pointing mode is initiated. Thirty seconds prior to the predicted time for the first target to come into view, the laser is energized and allowed to thermally stabilize.

The system then enters the acquisition mode, which requires a sequential search about the nominal pointing direction in a preprogrammed search sequence. The acquisition strategy selected for this experiment is a variable dwell approach. At each of the search locations, the received signal is compared to dual threshold criteria. The search would continue at each location until a positive no target or a positive target in view decision would be made. This strategy is maximally effective for the relatively slow moving SIPS platform.

Once the target is detected, a short dither sequence is performed to estimate the true target angular location and the system enters the ranging mode.

In the ranging mode, the SIPS is commanded to point toward each of the targets in the grid in a preprogrammed sequence. At each discrete target, ranging measurements are made for n successive pulses, where n is a variable dependant on the size of the grid and the angular agility of the SIPS. Each measurement group is tested to ensure validity, and used to update the state vector using a Kalman filter technique. This sequence continues until the last target in the grid is no longer in view. A typical ranging pass lasts for about 260 seconds.

The post-pass mode is provided to ensure an orderly power down sequence and completion of all buffered functions. The post pass mode is completed when the last of the LRS components is powered down and the SIPS returns to the cold soak mode.

3.2.2 Opto-Mechanical Subsystem

The OMS is virtually identical to the subsystem described for the pallet mounted experiment. The major difference is that it is not necessary to fold the transmit beam to the receiver centerline, since the gimbaled mirror is not required. The laser output is passed through a frequency doubler and then expanded to approximately 2 cm diameter. The frequency doubled output is reflected by a dichroic mirror, and delivered to the output lens, which adjusts the beam to a 0.5 mrad divergence. The beam also passes through a partially reflective mirror which extracts approximately one percent of the pulse energy. This portion of the pulse energy is passed through a neutral density filter and reflected to the ranging detector for a departure time measurement.

The receive optics path includes a bifurcating mirror, which allows a star sensor to be used to calibrate the receiver boresight. The central portion of the receiver field of view passes through the bifurcating mirror, and encounters two torque motor beam steering components, a narrow band filter, and a variable neutral density filter. Another bifurcating mirror limits the high speed detector field of view to ~ 0.5 mrad, and reflects the balance of the field of view to a quadrant photomultiplier.

Various alternatives for on-orbit calibration of transmitter and receiver alignment were considered. The most appropriate technique was judged to be performed in conjunction with a cooperative ground station. The ground station would illuminate the Shuttle. The QPMT output is used to determine the precise calibration of receiver boresight to ARS baseplate. The LRS transmitter output would be detected by a cluster of calibrated receivers on the ground, and a boresight error determined. This data would then be transmitted to the LRS via normal Shuttle communications, and used to bias the computed pointing angles.

A television camera could be added to enhance the capability of the experiment operator to monitor and control the experiment. The collecting power and resolution of the receiver telescope would allow both the star sensor and the television camera to share (50-50 division) the outer portion of the receiver field of view. Alternatively, a torque motor beam steering element could be employed to select either the star sensor or the television camera.

3.2.3 Interfaces

The experiment interfaces are with the SIPS, and ultimately with the Shuttle/Spacelab. The LRS optical bench is attached to the canister gimbal frame and thermally shielded as much as possible. Those components not mounted on the optical bench are attached to a support structure behind the optical bench.

The LRS data interface with the SIPS and the Shuttle/Spacelab is through a standard RAU. As presently planned, data will be transferred to the Shuttle for storage and subsequent transmission to the ground, in the same manner as for the pallet mounted experiment, and will not be presented again in this section. The total data load through the RAU will, however, be somewhat greater for this experiment, due to the need to transmit pointing commands to the SIPS computer and to receive gimbal angle data necessary to eliminate the time varying range bias.

The electrical power budget shown in Section 3.1.4 is the same except for deletion of the thermal control subsystem (provided by the SIPS). The energy budget, however, is considerably reduced since the clock will not be operated continuously. Table 11 summarizes the energy consumption for this experiment. The prime power requirement for the power conditioner has been adjusted to account for deletion of the active thermal control subsystem.

TABLE 11
POWER PROFILE ESTIMATE

	POWER VS MODE			ENERGY CONSUMED (10 PASSES) (KW-HRS)
	ALIGN (45 MIN.)	PASS (280 SEC)	POST-PASS (5 MIN)	
LASER PWR SUPPLY	0	390W	0	0.3
ARS SENSORS	66W	66W	66W	0.6
POWER CONDITIONER	137W	137W	137W	1.25
TOTAL	203W	593W	203W	2.15

4. Conclusions

A laser ranging experiment has been defined and analyzed, and two laser ranging systems have been configured to accomplish the experiment. The experiment has been shown to be capable of meeting the target grid reconstruction accuracy goal. A number of important factors have been identified and documented, such as the importance of widely spaced out-rigger targets. The prototype system used to validate the experiment design was a one pps laser ranging system with a 10 cm rms accuracy, with relatively modest slew rate requirements. The two laser ranging systems employ a 10 pps laser, with rms accuracies in the 2 to 10 cm range, depending on environmental considerations. The pallet mounted experiment has considerably greater angular agility than the SIPS mounted experiment, since only the gimballed folding mirror must be moved. Both systems are judged capable of meeting the assumed experiment objectives of determining the relative positions of five targets with respect to each other with an overall three dimensional reconstruction accuracy better than 2 cm rms; the pallet mounted experiment is capable of accomplishing more ambitious experiments.

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APPENDIX A

Coupling of geopotential terms with spacecraft motion.

Let X be spacecraft position

v be spacecraft velocity

and h be geopotential terms.

Then the equations of motion are

$$\begin{aligned}\dot{X} &= v \\ \dot{v} &= a(X, h) \\ \dot{h} &= 0\end{aligned}$$

where a represents the total acceleration acting on the spacecraft. Then

$$F = \frac{\Delta}{\begin{pmatrix} v \\ a \\ 0 \\ X \\ v \\ h \end{pmatrix}} = \begin{pmatrix} 0 & I & 0 \\ \frac{\partial a}{\partial X} & 0 & \frac{\partial a}{\partial h} \\ 0 & 0 & 0 \end{pmatrix}$$

and the state transition matrix is

$$e^{\int_0^{\Delta t} F(\tau) d\tau} = \begin{pmatrix} \phi & \psi \\ 0 & I \end{pmatrix}$$

$$e^{\int_0^{\Delta t} \begin{pmatrix} 0 & I \\ \frac{\partial a}{\partial X} & a \end{pmatrix} d\tau}$$

Here $\phi = e^{\int_0^{\Delta t} \begin{pmatrix} 0 & I \\ \frac{\partial a}{\partial X} & a \end{pmatrix} d\tau}$ is the usual orbit state transition matrix. To obtain ψ note that ϕ and ψ satisfy

$$\begin{pmatrix} \dot{\phi} & \dot{\psi} \\ 0 & I \end{pmatrix} = \begin{pmatrix} 0 & I & 0 \\ \frac{\partial a}{\partial X} & 0 & \frac{\partial a}{\partial h} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi & \psi \\ 0 & I \end{pmatrix} \text{ with } \begin{pmatrix} \phi & \psi \\ 0 & I \end{pmatrix}_0 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

This gives

$$\dot{\phi} = \begin{pmatrix} 0 & I \\ \frac{\partial a}{\partial v} & a \end{pmatrix} \phi, \quad \phi(0) = I$$

$$\dot{\psi} = \begin{pmatrix} 0 & I \\ \frac{\partial a}{\partial X} & a \end{pmatrix} \psi + \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix}, \quad \psi(0) = 0$$

giving

$$\psi(t) = \phi(t) \int_0^t \phi^{-1}(\tau) \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} d\tau$$

We assume $\frac{\partial a}{\partial h}$ to be constant (i.e., the geopotentials do not change appreciably over the state transition time interval). If we cannot do this then numerical integration would have to be resorted to and this we want to avoid. This assumption allows $\partial a / \partial h$ to be factored from the integrand

$$\begin{aligned} \psi(t) &= \phi(t) \int_0^t \phi^{-1}(\tau) d\tau \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} \\ &= \int_0^t \phi(t) \phi^{-1}(\tau) d\tau \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} \\ &= \int_0^t \phi(t) \phi(-\tau) d\tau \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} \\ &= \int_0^t \phi(t-\tau) d\tau \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} \\ &= \int_t^0 \phi(\zeta) d\zeta \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} \\ &= \int_0^t \phi(\zeta) d\zeta \begin{pmatrix} 0 \\ \frac{\partial a}{\partial h} \end{pmatrix} \end{aligned}$$

so that coupling terms can be obtained by simple quadratures from the spacecraft state transition.

APPENDIX B

POINTING COVARIANCE

Following is the derivation of an expression for the variance of the pointing uncertainty for directing an instrument on the spacecraft with position vector $\bar{r}$ towards a reflector at p . Let $\bar{\rho}$ be the vector from F to $\bar{p}$ then

$$\bar{\rho} = \bar{p} - \bar{r} \quad (1)$$

Let $\bar{e}_\rho$ be a unit vector in the direction of $\bar{\rho}$ and denote vector magnitude by omission of overbars. We then have

$$\bar{\rho} = \rho \bar{e}_\rho \quad (2)$$

It is desired to express the error in angular orientation of $\bar{\rho}$ due to the errors in $\bar{r}$ and $\bar{p}$. The covariances of F and $\bar{p}$ are obtainable from the total state covariance matrix. The angular variation is given by a variation in $\bar{e}_\rho$. From (2)

$$\delta \bar{\rho} = \delta_\rho \bar{e}_\rho + \rho \delta \bar{e}_\rho \quad (3)$$

But

$$\rho^2 = \bar{\rho} \cdot \bar{\rho} \quad (4)$$

giving

$$2\rho \delta \rho = 2\bar{\rho} \cdot \delta \bar{\rho} \quad (5)$$

Inserting (5) into (3)

$$\delta \bar{\rho} = \left(\frac{\bar{\rho}}{\rho} \cdot \delta \bar{\rho} \right) \bar{e}_\rho + \rho \delta \bar{e}_\rho \quad (6)$$

or using (2)

$$\rho \delta \bar{e}_\rho = \delta \bar{\rho} - (\bar{\rho}_\rho \cdot \delta \bar{\rho}) \bar{e}_\rho \quad (7)$$

Taking the dot produced of (7) with itself

$$\begin{aligned} \rho^2 \delta \bar{e}_\rho \cdot \delta \bar{e}_\rho &= \delta \bar{\rho} \cdot \delta \bar{\rho} + (\bar{e}_\rho \cdot \delta \bar{\rho})^2 \\ &\quad - 2(\bar{e}_\rho \cdot \delta \bar{\rho})^2 \\ &= \delta \bar{\rho} \cdot \delta \bar{\rho} - (\bar{e}_\rho \cdot \delta \bar{\rho})(\delta \bar{\rho} \cdot \bar{e}_\rho) \end{aligned} \quad (8)$$

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The angular variance of interest is $\sigma^2 = E(\delta \bar{\rho}_\rho \cdot \delta \bar{\rho}_\rho)$ so taking expectations of (8) we obtain

$$\begin{aligned} \rho^2 \sigma^2 &= E(\delta \bar{p} - \delta \bar{r}) \cdot (\delta \bar{p} - \delta \bar{r}) \\ &\quad - \bar{\rho}_p \cdot E((\delta \bar{p} - \delta \bar{r})(\delta \bar{p} - \delta \bar{r})) \cdot \bar{\rho}_\rho \end{aligned} \quad (9)$$

where the central factors of the last term is a dyadic. We let the 3×3 matrix A represent this dyadic then

$$A = S_{pp} - S_{pr} - S_{rp} + S_{rr} \quad (10)$$

where S_{pp} is the covariance of the reflector position components, S_{rr} is the covariance of spacecraft position and $S_{pr} = S_{rp}^T$ is the correlation between spacecraft and reflector position (induced by the estimation procedure).

Expression (9) becomes

$$\sigma^2 = \frac{1}{\rho^2} (\text{tr } A - e^T A e) \quad (11)$$

where $\text{tr } A$ denotes the trace of A (i.e., $\text{tr } A = A_{11} + A_{22} + A_{33}$) and e is a 3×1 column vector representing to vector $\bar{e}_p$ (i.e., e contains the direction cosines of $\bar{\rho}$). Expression (11) is implemented in a subroutine.

APPENDIX C

Suppose that one observation is made on reflector #1 from S and subsequently one observation on reflector #2. Assume that the geometry of the observer reflector configuration does not change between observations and that the observations are of range with a random error having variance $Q = \sigma_m^2$ and a systematic error having a variance $P_b = \sigma_b^2$. We wish to find the 3×3 covariance matrix P_ρ of the differential position of reflector #1 with respect to reflector #2.

Let the state vector be

$$10 \times 1 \quad X = \begin{pmatrix} 3 \text{ coordinates of reflector \#1} \\ 3 \text{ coordinates of reflector \#2} \\ 3 \text{ coordinates of observer} \\ \text{Systematic error in measurements} \end{pmatrix} = \begin{pmatrix} 3 \times 1 \\ r_1 \\ 3 \times 1 \\ r_2 \\ 3 \times 1 \\ r_S \\ 1 \times 1 \\ r_b \end{pmatrix} \quad \begin{matrix} \\ \\ \\ \\ \\ \\ 3 \times 3 \end{matrix}$$

Let both reflectors have an apriori position error covariance P_T and the observer position error 3×3 P_S , then if these are uncorrelated the apriori covariance of the state is

$$10 \times 10 \quad P_o = \begin{pmatrix} P_T & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_S & 0 \\ 0 & 0 & 0 & P_b \end{pmatrix} \quad (1)$$

For a measurement of range y_1 from observer to reflector #1 we have y_1 determined by the expression

$$(y_1 - b)^2 = (r_1 - S)^T(r_1 - S)$$

where the superscript T denotes transpose of a matrix. Taking variations in this we obtain

$$2(y_1 - b)(\delta y_1 - \delta b) = 2(r_1 - S)^T(\delta r_1 - \delta S)$$

giving

$$\delta y_1 = \frac{(r_1 - S)^T}{y_1 - b} (\delta r_1 - \delta S) + \delta b$$

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Let a unit vector in the direction of reflector #1 from the observer be denoted by e_1 and similarly e_2 for reflector #2. Then

$$e_1 = \frac{r_1 - S}{Y_1 - b}$$

and

$$\begin{aligned} \delta Y_1 &= e_1^T (\delta r_1 - \delta S) + \delta b \\ &= \begin{pmatrix} 1 \times 3 & 1 \times 3 & 1 \times 3 & 1 \times 1 \end{pmatrix} \begin{pmatrix} \delta r_1 \\ \delta r_2 \\ \delta S \\ \delta b \end{pmatrix} \\ &= (e_1^T, 0, -e_1^T, 1) \delta X \\ &= (e_1^T, 0, -e_1^T) \delta X \end{aligned}$$

Denote

$$M_1^{1 \times 10} \triangleq (e_1^T, 0, -e_1^T, 1) = \frac{\partial Y}{\partial X} \quad (2)$$

This will be needed in to determine the state covariance after the observation.

The Kalman update formula is

$$P_1^{10 \times 10} = P_0^{10 \times 10} - P_0^{10 \times 10} M_1^{10 \times 1} (M_1^{10 \times 1} P_0^{10 \times 10} M_1^{1 \times 10} + Q)^{-1} M_1^{1 \times 10} P_0^{10 \times 10} \quad (3)$$

where P_0 is the state covariance after incorporation of the observation on reflector #1.

$$\text{Define } Z_1^{1 \times 1} = M_1^{1 \times 10} P_0^{10 \times 10} M_1^{10 \times 1} + Q \quad (4)$$

Now from (1) and (2)

$$P_0^{10 \times 10} M_1^{1 \times 10} = \begin{pmatrix} P_T & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_S & 0 \\ 0 & 0 & 0 & P_b \end{pmatrix} \begin{pmatrix} e_1 \\ 0 \\ -e_1 \\ 1 \end{pmatrix} = \begin{pmatrix} P_T e_1 \\ 0 \\ -P_S e_1 \\ P_b \end{pmatrix} \quad (5)$$

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and

$$\begin{aligned}
 Z_1 &= (e_1^T, 0, -e_1^T, 1) \begin{pmatrix} P_T e_1 \\ 0 \\ -P_S e_1 \\ P_b \end{pmatrix} + Q \\
 &= e_1^T P_T e_1 + e_1^T P_S e_1 + P_b + Q \\
 &= e_1^T (P_T + P_S) e_1 + P_b + Q \quad (6)
 \end{aligned}$$

and from (3) (factoring out the scalar Z_1^{-1} using (5) and noting that P_T, P_S, P_b are symmetric

$$\begin{aligned}
 P_1 &= \begin{pmatrix} P_T & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_S & 0 \\ 0 & 0 & 0 & P_b \end{pmatrix} - \frac{1}{Z_1} \begin{pmatrix} P_T e_1 \\ 0 \\ -P_S e_1 \\ P_b \end{pmatrix} (e_1^T P_T, 0, -e_1^T P_S, P_b) \\
 P_1 &= \begin{pmatrix} P_T - 1/Z_1 P_T e_1 e_1^T P_T & 0 & 1/Z_1 P_T e_1 e_1^T P_S & -1/Z_1 P_T e_1 P_b \\ 0 & P_T & 0 & 0 \\ 1/Z_1 P_S e_1 e_1^T P_T & 0 & P_S - 1/Z_1 P_S e_1 e_1^T P_S & 1/Z_1 P_S e_1 P_b \\ -1/Z_1 P_b e_1^T P_T & 0 & 1/Z_1 P_b e_1^T P_S & P_b - 1/Z_1 P_b P_b \end{pmatrix} \quad (7)
 \end{aligned}$$

For the observation of reflector #2 we have

$$\begin{aligned}
 1 \times 10 \\
 M_2 &= (0, e_2^T, -e_2^T, 1) \quad (8)
 \end{aligned}$$

and

$$\begin{aligned}
 10 \times 10 \\
 P_2 &= P_1 - \frac{1}{Z_2} P_1 M_2^T M_2 P_1, \quad (9)
 \end{aligned}$$

with

$$\begin{aligned}
 1 \times 1 \\
 Z_2 &= M_2 P_1 M_2 + Q \quad (10)
 \end{aligned}$$

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Proceeding as before

$$P_1 M_2^T = \begin{pmatrix} -\frac{1}{Z_1} P_T e_1 (e_1^T P_S e_2 + P_b) \\ P_T e_2 \\ -P_S e_2 + \frac{1}{Z_1} P_S e_1 (e_1^T P_S e_2 + P_b) \\ P_b - \frac{1}{Z_1} P_b (e_1^T P_S e_2 + P_b) \end{pmatrix}$$

$$\begin{aligned} Z_2 &= e_2^T P_T e_2 + e_2^T P_S e_2 - \frac{1}{Z_1} e_2^T P_S e_1 (e_1^T P_S e_2 + P_b) \\ &\quad + P_b - \frac{1}{Z_1} P_b (e_1^T P_S e_2 + P_b) + Q \\ &= e_2^T (P_T + P_S) e_2 + P_b + Q - \frac{1}{Z_1} (e_2^T P_S e_1 + P_b) (e_1^T P_S e_2 + P_b) \end{aligned} \quad (11)$$

We are not directly interested in the covariance P_2 of the entire state vector but rather in the covariance of the separation p of the reflectors. Let

$$\begin{matrix} 3 \times 1 & 3 \times 1 & 3 \times 1 \\ \rho & = & r_2 - r_1 \end{matrix}$$

then

$$\frac{\partial \rho}{\partial X} = H = \begin{matrix} 3 \times 10 & 3 \times 3 & 3 \times 3 & 3 \times 3 & 3 \times 1 \\ (-I, & I, & 0, & 0) \end{matrix} \quad (12)$$

$$\text{where } I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The covariance of the error in ρ after both observations is

$$\begin{aligned} \begin{matrix} 3 \times 3 & 3 \times 10 & 10 \times 10 & 10 \times 3 \\ P_\rho & = & H & P_2 & H^T \\ & = & H P_1 H^T - 1/Z_2 & H P_1 M_2^T M_2 P_1 H^T \end{matrix} \end{aligned}$$

by referring to (9), to evaluate this expression we will do it piece by piece as follows.

From (7) and (12)

$$HP_1 H^T = 2P_T - \left(1/Z_1\right) P_T e_1 e_1^T P_T$$

and from (7), (12) and (8)

$$HP_1 M_2^T = P_T e_2 + \frac{1}{Z_1} P_T e_1 (e_1^T P_S e_2 + P_b)$$

Inserting these into (13) we obtain

$$\begin{aligned} P_\rho &= 2P_T - \frac{1}{Z_1} P_T e_1 e_1^T P_T \\ &\quad - \frac{1}{Z_2} (P_T e_2 + \frac{1}{Z_1} P_T e_1 (e_1^T P_S e_2 + P_b)) (e_2^T P_T + \frac{1}{Z_1} (e_2^T P_S e_1 + P_b) e_1^T P_T) \\ P_\rho &= 2P_T - \frac{1}{Z_1} P_T e_1 e_1^T P_T \\ &\quad - \frac{1}{Z_2} P_T (e_2 + \frac{1}{Z_1} e_1 (e_1^T P_S e_2 + P_b)) (e_2^T + \frac{1}{Z_1} (e_2^T P_S e_1 + P_b) e_1^T) P_T \end{aligned} \quad (14)$$

Let $1 \times 1 \Delta$
 $Z_S = e_1^T P_S e_2 + P_b$ (15)

then

$$\begin{aligned} P_\rho &= 2P_T - \frac{1}{Z_1} P_T e_1 e_1^T P_T \\ &\quad - \frac{1}{Z_2} P_T (e_2 + \frac{Z_S}{Z_1} e_1) (e_2^T + \frac{Z_S}{Z_1} e_1^T) P_T \end{aligned} \quad (16)$$

Define $Z_{10} = Z_1 = e_1^T (P_T + P_S) e_1 + P_b + Q$ (17)

$Z_{20} = e_2^T (P_T + P_S) e_2 + P_b + Q$ (18)

then

$$Z_2 = Z_{20} - \frac{Z_S^2}{Z_{10}} \quad (19)$$

by referring to (11).

A factor appearing in the middle of the last two terms of formula (16) is

$$\begin{aligned} & \frac{1}{z_1} e_1 e_1^T + \frac{1}{z_2} (e_2 + \frac{z_S}{z_1} e_1) (e_2^T + \frac{z_S}{z_1} e_1^T) \\ &= \frac{1}{z_{10}} e_1 e_1^T + \frac{1}{z_{20} - \frac{z_S}{z_{10}}} (e_2 + \frac{z_S}{z_{10}} e_1) (e_2^T + \frac{z_S}{z_{10}} e_1^T) \\ &= \left(\frac{1}{z_{10}} + \frac{z_S^2}{z_{10}^2} \frac{1}{z_{20} - \frac{z_S}{z_{10}}} \right) e_1 e_1^T + \frac{1}{z_{20} - \frac{z_S}{z_{10}}} e_2 e_2^T + \frac{z_S}{z_{10}} \frac{1}{z_{20} - \frac{z_S}{z_{10}}} (e_2 e_1^T + e_1 e_2^T) \\ &= \frac{z_{20}}{z_{10} z_{20} - z_S^2} e_1 e_1^T + \frac{z_{10}}{z_{10} z_{20} - z_S^2} e_2 e_2^T + \frac{z_S}{z_{10} z_{20} - z_S^2} (e_2 e_1^T + e_1 e_2^T) \end{aligned}$$

so that the formulas (16), (15), (17) and (18) have the form which, as expected, is symmetric with respect to the two measurements

$$\begin{aligned} P_\rho &= 2P_T - P_T \frac{z_{20} e_1 e_1^T + z_{10} e_2 e_2^T + z_S (e_2 e_1^T + e_1 e_2^T)}{z_{10} z_{20} - z_S^2} P_T \\ z_S &= e_1^T P_S e_2 + P_b \\ z_{10} &= e_1^T (P_T + P_S) e_1 + P_b + Q \\ z_{20} &= e_2^T (P_T + P_S) e_2 + P_b + Q \end{aligned} \tag{20}$$

Special case: Both reflectors with same direction from the observer.

In this case $e_1 = e_2 \stackrel{\Delta}{=} e$

From (20)

$$z_{10} = z_{20} \stackrel{\Delta}{=} z_0$$

and

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$$P_{\rho} = 2P_T - P_T \frac{2Z_0 + 2Z_S}{Z_0^2 - Z_S^2} ee^T P_T$$

$$= 2P_T - 2P_T \frac{ee^T}{Z_0 - Z_S} P_T$$

with $Z_0 - Z_S = (e^T(P_T + P_S)e + P_b + Q) - (e^T P_S e + P_b)$

$$= e^T P_T e + Q$$

The minimum eigenvalue of P_e must obviously correspond to the direction of e and is given in this case by

$$e^T P_{\rho} e = 2 \frac{e^T P_T e}{e^T P_T e + Q} Q$$

since $Q = \sigma_m^2$ and $\frac{e^T P_T e}{e^T P_T e + Q} < 1$

We have that the standard deviation of the position difference in the direction of the measurements is

$$< \sqrt{2} \sigma_m$$

as suspected.

Suppose the reflector directions are separated by an angle 2δ . Set up a coordinate system with x axis bisecting the directions to the targets with the y axis in the plane of the observer and both reflectors. The unit vectors e_1 and e_2 are

$$e_1 = \begin{pmatrix} \cos \delta \\ \sin \delta \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} \cos \delta \\ -\sin \delta \\ 0 \end{pmatrix}$$

Denote the unit vector in the direction of the x -axis by e

$$e = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Assume for simplicity, that in this coordinate system that all of the aprior position covariances are diagonal. Thus,

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$$P_T = \begin{pmatrix} \sigma_{Tx}^2 & 0 & 0 \\ 0 & \sigma_{Ty}^2 & 0 \\ 0 & 0 & \sigma_{Tz}^2 \end{pmatrix}, P_S = \begin{pmatrix} \sigma_{Sx}^2 & 0 & 0 \\ 0 & \sigma_{Sy}^2 & 0 \\ 0 & 0 & \sigma_{Sz}^2 \end{pmatrix}$$

$$Q = \sigma_m^2,$$

$$P_b = \sigma_b^2$$

Referring to (20),

$$Z_S = \sigma_{Sx}^2 \cos^2 \delta - \sigma_{Sy}^2 \sin^2 \delta + \sigma_b^2$$

$$Z_0 \triangleq Z_{10} = Z_{20} = (\sigma_{Tx}^2 + \sigma_{Sx}^2) \cos^2 \delta + (\sigma_{Ty}^2 + \sigma_{Sy}^2) \sin^2 \delta + \sigma_b^2 + \sigma_m^2$$

From symmetry we know that the minimum eigenvalue of P_ρ corresponds to the direction e so we wish to evaluate $e^T P_\rho e$.

$$\text{Now } e^T P_T e = \sigma_{Tx}^2, e^T P_T e_1 = e_1^T P_T e = \sigma_{Tx}^2 \cos \delta$$

and

$$\begin{aligned} e^T P_\rho e &= 2\sigma_{Tx}^2 - \frac{Z_{20}\sigma_{Tx}^4 \cos^2 \delta + Z_{10}\sigma_{Tx}^4 \cos \delta + Z_S 2\sigma_{Tx}^4 \cos^2 \delta}{Z_{10}Z_{20} - Z_S^2} \\ &= 2\sigma_{Tx}^2 - 2\sigma_{Tx}^4 \cos^2 \delta \frac{Z_0 + Z_S}{Z_0^2 - Z_S^2} \\ &= \frac{2\sigma_{Tx}^2}{Z_0 - Z_S} [Z_0 - Z_S - \sigma_{Tx}^2 \cos^2 \delta] \\ &= 2\sigma_{Tx}^2 \frac{(\sigma_{Ty}^2 + 2\sigma_{Sy}^2) \sin^2 \delta + \sigma_m^2}{\sigma_{Tx}^2 \cos^2 \delta + (\sigma_{Ty}^2 + 2\sigma_{Sy}^2) \sin^2 \delta + \sigma_m^2} \end{aligned}$$

This is the desired formula. It shows how the separation angle δ allows the observer position variance (σ_{Sy}^2) (normal to the mean line of sight and in the plane of the observer and targets) to degrade the experiment since $\sigma_{Sy}^2 \sin^2 \delta$ can easily be of the same order of magnitude as σ_m^2 .